MATH 53 WORKSHEET, FEBRUARY 10TH - SOLUTION

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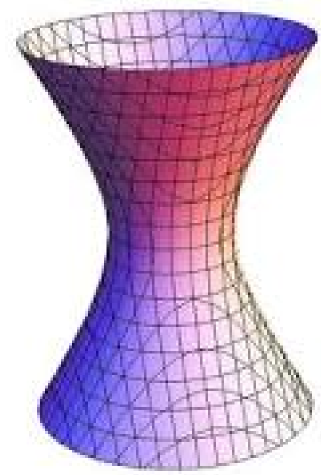
(1) Consider the equation $x^2 + y^2 - z^2 = 1$

a) Draw the traces of this surface in the xy-plane, the yz-plane, and the xz-plane.

Solution : xy trace: circle; yz trace: hyperbola opening along y-axis; xz trace: hyperbola opening along x-axis.

b) Now draw the surface in 3D space.

Solution :

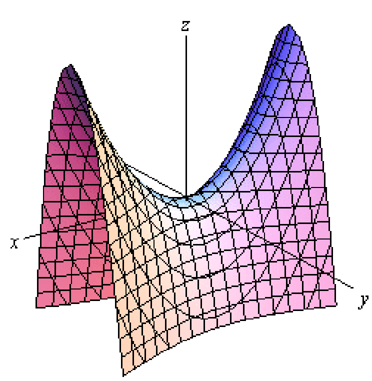


(2) Consider the parametrized space curve $x = \cos t$, $y = \sin t$, $z = \cos 2t$.

(a) Show that the curve lies on the surface $z = x^2 - y^2$.
Solution: The left hand side is $\cos 2t$; the right hand side is $\cos^2 t - \sin^2 t$. These are the same by some trig identity.

(b) Show that the curve lies also on the surface $x^2 + y^2 = 1$.
Solution: The left hand side is $\cos^2 t + \sin^2 t$; the right hand side is 1. These are the same by the genius of Pythagoras.

(c) Draw the two surfaces, and try to see where they intersect - this is a picture of the parametrized curve.
The first surface is the hyperbolic paraboloid:



Now imagine cutting this with a vertical cylinder (the second surface). This gives the curve which is pictured in Figure I of problems 21-26 on page 846 of your book (also see attached image in problem 4, below).

(3) Find an equation of the plane which contains the point $(1,1,1)$ and is perpendicular to the line parametrized by $x = 2 + 3t$, $y = 7t - 1$, $z = 2 - 2t$.

Solution: To describe any plane, we want to write an equation like $ax + by + cz = d$, and we just need to find a, b, c, d. We can always find a, b, c by finding a normal vector to the plane (any vector whose direction is perpendicular to the plane). Then we find d by plugging in any point on the plane. Since we already have the point (it's given to us, namely (1,1,1)), we only need to find a normal vector.

Now let's try to find this normal vector. We know that the line $\langle 2+3t, 7t-1, 2-2t\rangle$ is perpendicular to our plane. Every line can be thought of as having a "start point" (when $t = 0$), and a direction vector. The start point is just the numbers without t next to them - in this case (2,-1,2). We don't need that here. For this problem, we need the direction vector, since that's the thing perpendicular to the plane. The direction vector is just the coefficients of t in the parametrization - in this case $\langle 3, 7, -2\rangle$.

So the direction vector of the line is $\langle 3, 7, -2\rangle$, and this is a normal vector to the plane. So a, b, c are 3,7, and -2, respectively. Thus far, the equation of the plane is

$$3x + 7y - 2z = d,$$

and we just need to find d. Since the point $(1,1,1)$ lies on the plane, we can plug it in:

$$3 \cdot 1 + 7 \cdot 1 - 2 \cdot 1 = d,$$

so $d = 8$. Thus the equation of the plane is

$$3x + 7y - 2z = 8$$

(4) Parametrize the curve that is the intersection of the surfaces $x^2 + y^2 = 4$ and $z = xy$.

Solution: I usually pick the equation with only two variables first and try to parametrize those two variables first. So let's look first at $x^2 + y^2 = 4$. This means that the x and y coordinates of the curve live on a circle, which we can parametrize as $x = 2\cos t$, $y = 2\sin t$ (there are other parametrizations that work, too). Now we use the other equation to find z:

$$z = xy = (2\cos t)(2\sin t) = 2\sin 2t,$$

(I used a double angle identity at the end just to be fancy, but it wasn't necessary). Thus our parametrization of the curve of intersection is

$$\mathbf{r}(t) = \langle 2\cos t, 2\sin t, 2\sin 2t\rangle.$$

In case you care, this curve is very similar to the curve of problem 2. Here's a picture of a picture of some chump's physics HW that I found online. This I believe to be the curve of problem 2, but the curve here in problem of 4 is basically the same...