Mathematics 105– Second Course in Analysis – Spring 2004

Instructor: Michael Christ. 809 Evans Hall, mchrist@math.berkeley.edu, 642-2143.
Office hours: M 12:00-1:00; W 1:00-2:00 and W 3:00-4:00.
Course web page: http://math.berkeley.edu/∼mchrist/Math105/homepage.html.
Lectures: MWF 2:10-3:00 in 85 Evans Hall.
Prerequisite: Math 104.
Required Work: Final and two midterm exams (Wednesday 2/18 and Friday 4/2), weekly problem sets due in class on Fridays.

Texts: (i) *Calculus on manifolds* by M. Spivak. (W.A. Benjamin, 1965)
(ii) *A concise introduction to the theory of integration* by D. Stroock. (third edition, Birkhäuser; ISBN 0-8176-4073-8)

Topics: **(I)** Multi-variable differential calculus (following Spivak). Derivatives and chain rule for functions of several variables. Implicit and inverse function theorems. **(II)** Measure and integration (following Stroock). Lebesgue measure on the real line. The Lebesgue integral. More general measures and integration. Convergence theorems. Multidimensional Lebesgue measure; Fubini's theorem. Lebesgue spaces, especially the square integrable case. Brief application to Fourier series. The divergence and Green theorems. Change of variables in multiple integrals.

The first part of the course (approximately 10 lectures)) will be a rigorous treatment of aspects of calculus in several variables. We'll begin with a review of the definition of derivative for functions beween Euclidean spaces and will formulate and prove the chain rule. This leads up to two big results, the implicit and inverse function theorems, which describe when a mapping between equidimensional spaces is locally invertible, and describes the set of all solutions of to an equation when the dimensions are unequal.

The second part will develop the theory of Lebesgue integration. Lebesgue's machinery permits the integration of much more general functions than does Riemann's, and provides superior tools for working with limits, even for Riemann integrable functions. It furnishes the underlying vocabulary and conceptual foundation for probability theory. It provides a framework for the most fundamental fact of Fourier analysis. Lebesgue integration is so fundamental that a special case of it is taught to six year olds under the name "addition".

Lebesgue theory is the heart of any first-graduate analysis course. This undergraduate treatment will focus on the basics, will emphasize the Euclidean case, and will downplay more abstract aspects.