

Mathematics 105– Spring 2004

Problem Set 1

The first topic of this course is differentiation of functions of several variables, culminating in the inverse and implicit function theorems. Our test will be Chapters 1 and 2 of Spivak's *Calculus on Manifolds*. Chapter 1 is a review of basic material which is part of the syllabus for Math 104. We'll go through it fairly quickly. Then we'll move on to Chapter 2, whose first four sections review material that you learned on the computational level in multivariable calculus, Math 53, but presumably without proofs or perhaps even precise definitions. We'll fill these in. In the final two sections we come to the inverse and implicit function theorems, which will be proved in detail. This part of the course will take 9 or 10 lectures. The remainder of the semester will be devoted to Lebesgue integration, based on the text by Stroock.

Problems (all from Spivak's text) for Chapter 1 (**due Friday, January 30**):
2,4,9,13, 14,16,19,21,22, 23,27,29.

This long assignment is intended as a review of some of Math 104. The sooner you finish it, the sooner you can focus on the newer material in Chapter 2.

Comments and hints:

#8: answer part (c) in terms of your answer to part (a).

#22: Recall that the complement of an open set is closed; and use #21.

#29: We know already that f is bounded above; let $M = \sup_{x \in A} f(x)$. If the value M is not taken on at some point of A , consider the opencover consisting of the sets $\{x \in A : f(x) < M - \frac{1}{n}\}$ to reach a contradiction.