Many Cheerful Facts

Organizers: Yael Degany & Jason Ferguson

Friday, 2:10pm–3:00pm, 939 Evans

Apr. 16 **Shuchao Bi**, UC Berkeley

Many Cheerful Facts and Conjectures about Commuting Varieties

Abstract: The commuting variety of a reductive Lie algebra $\mathfrak{g}$ is defined as the reduced subscheme of $\mathfrak{g} \oplus \mathfrak{g}$ cut out by equations $[X, Y] = 0$. Geometrically, it is known to be irreducible, but algebraically even for $\mathfrak{g} = \mathfrak{gl}_n$ it is not known whether the ideal generated by these equations is radical (or equivalently prime), i.e., whether these equations serve to define the commuting variety as a scheme. It was conjectured by Artin and Hochster that the commuting variety is Cohen-Macaulay. In this talk, I will first present the proof the irreducibility and then discuss some recent results and methods related to the above open problem and conjecture.

For the first half of the talk, the only prerequisite is linear algebra and for the second half, people need to know a little bit of commutative algebra. Based on this, I think undergraduates are welcome.