Many Cheerful Facts

Organizers: Michael Pejic & Damien Mondragon

Tuesday, 2:10-3:00 pm, 939 Evans

May. 12 **Arthur Tilley**, UC Berkeley

The Banach-Tarski Paradox–Part II

The Bolyai-Gerwein Theorem says that two polygons are congruent by dissection if and only if they have the same area. In 1900, Dehn gave a negative solution to Hilbert's Third problem to conclude that a similar result does not hold in $\mathbb{R}^3$: Two polyhedra of equal volume are not necessarily congruent by dissection. A natural question then is, what if instead of dissecting a shape into polygons or polyhedra, we allow the pieces to be any subset? Say that two sets A and B in $\mathbb{R}^n$ are equidecomposable if they can be partitioned into $A_1, \ldots, A_n$ and $B_1, \ldots, B_n$ such that for isometries $g_1, \ldots, g_n$, we have $g_1(A_1) = B_1, \ldots, g_n(A_1) = B_n$. If μ is a finitely additive measure, invariant under isometries, is it the case that two sets that are equidecomposable have the same measure? Given the Axiom of Choice, the answer turns out to be an emphatic "no."

I will prove the Strong form of the Banach-Tarski theorem, which says that if A and B are any two bounded sets in $\mathbb{R}^3$, each with nonempty interior, then A and B are equidecomposable. As commonly illustrated, a solid ball the size of a pea may be taken apart into finitely many pieces that can be moved around to produce a solid ball the size of the sun.

Yeah, you should probably come.