# MANY CHEERFUL FACTS

## presents

## New Deductive Procedures

### a talk by Andre Kornell

**13:10 − 14:00** on Wednesday, February 20, in room 1015.

⭐⭐⭐ **PLEASE NOTE THE EXCEPTIONAL TIME.** ⭐⭐⭐

> *"... from a formalist point of view we may look at our theory syntactically and may consider that what we have done is to introduce new deductive procedures rather than new mathematical entities."*
>    -Abraham Robinson [a]

Non-standard analysis is rigorous analysis with infinitesimals. The use of infinitesimals was first justified by Abraham Robinson in the 1960's. Internal Set Theory (IST) is the approach to non-standard analysis developed by Edward Nelson. Rather than implementing infinitesimals by constructing an appropriate field extension of $\mathbb{R}$, internal set theory introduces infinitesimals by elevating the arguments involving such extensions to general mathematical principles.

The advantage of this approach to non-standard analysis is that it requires almost no background in logic to apply. However, the axioms of internal set theory are not only not obviously true, but are counter-intuitive and readily lead to a contradiction if applied carelessly. Why should we then trust these new mathematical principles?

I will present a proof that IST is a *conservative extension* of ZFC, the widely accepted foundation of mathematics. Thus, IST is an addition to ordinary mathematical reasoning that relies on no new assumptions. Furthermore, this can be shown *finitistically*! Those who accept the methods of modern mathematics in their entirety have nothing to fear in adding IST to their mathematical tool box.

Rather than present the proof in full detail, I will use it to introduce a number of beautiful notions from mathematical logic. Throughout the talk I will emphasize connections and parallels with other areas of mathematics. I'll also try to say something about formalism and ultrafinitism in a half-hearted attempt to convince you that the axioms of IST might make some sense after all. If sufficient time remains we will prove every theorem taught in Math 1A.

*No background in mathematical logic will be assumed.*

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[a]Abraham Robinson, *Non-Standard Analysis*, Revised Edition, American Elsevier, New York, 1974

> *I am the very model of a modern Major General,*
> *I've information vegetable, animal, and mineral,*
> *I know the kings of England, and I quote the fights historical*
> *From Marathon to Waterloo, in order categorical;*
> *I'm very well acquainted, too, with matters mathematical,*
> *I understand equations, both the simple and quadratical,*
> *About binomial theorem I'm teeming with a lot o' news,*
> *With many cheerful facts about the square of the hypotenuse!*
>
>    — Gilbert & Sullivan, $P \circ P$

The website for Many Cheerful Facts is
`http://math.berkeley.edu/~mcf/`