

MANY CHEERFUL FACTS

presents

A Total Order on Chaos

a talk by Justin Blanchard

12:10 – 13:00 on Wednesday, January 30, in room 1015.

Repeated iteration of a map can produce chaotic behavior and extreme sensitivity to initial conditions. Iterating even the simplest maps can lead to a rich and interesting theory within the field of dynamical systems. Iterating rather complicated maps can produce passable models for physical or financial situations, which frustrate the forecasters who want to use them by exhibiting similar phenomena. This talk is about the former approach. Specifically, we'll iterate continuous maps from an interval to itself.

In 1964, the Ukrainian mathematician Sharkovsky completely described the coexistence of cycles of different periods under such a map. The preorder “ $m \prec n$ if an m -periodic point implies an n -periodic point” is in fact the following total ordering of \mathbb{N} :

$$\begin{aligned} 2^0 \cdot 3 \prec 2^0 \cdot 5 \prec 2^0 \cdot 7 \prec 2^0 \cdot 9 \prec \dots \prec \\ 2^1 \cdot 3 \prec 2^1 \cdot 5 \prec 2^1 \cdot 7 \prec 2^1 \cdot 9 \prec \dots \prec \\ 2^2 \cdot 3 \prec 2^2 \cdot 5 \prec 2^2 \cdot 7 \prec 2^2 \cdot 9 \prec \dots \prec \\ \dots \prec \\ \dots \prec 2^3 \prec 2^2 \prec 2^1 \prec 2^0 \end{aligned}$$

A decade later, the rediscovery of a special case (“Period three implies chaos”, Li and Yorke) opened the wider world’s eyes to this cute, long-overlooked result. Today, the theorem is a standard part of a first course on dynamical systems or “mathematical chaos”. One day, some spelling of its author’s name (Шарковський) in the Latin alphabet, or some symbol for the above ordering (\prec , \succ , or \triangleright), might also be standard.

The goals of this talk are to justify the Sharkovsky ordering with some simple proofs, and to sketch examples of functions with each set of cycle periods allowed by the theorem. The speaker will expect the audience to know very little about dynamical systems, and vice versa.

*I am the very model of a modern Major General,
I've information vegetable, animal, and mineral,
I know the kings of England, and I quote the fights historical
From Marathon to Waterloo, in order categorical;
I'm very well acquainted, too, with matters mathematical,
I understand equations, both the simple and quadratical,
About binomial theorem I'm teeming with a lot o' news,
With many cheerful facts about the square of the hypotenuse!*

— Gilbert & Sullivan, $P \circ P$

The website for Many Cheerful Facts is
<http://www.math.berkeley.edu/~mcf>