

MANY CHEERFUL FACTS

presents

Gödel's Theorems

a talk by Kenny Easwaran

1:10 - 2:00pm on Thursday, September 6, in room 1015.

At the beginning of the 20th century, David Hilbert set out a project to answer all mathematical questions. The development of formal logic between 1870 and 1920 convinced him that the notion of a rigorous proof could be properly formalized. With this notion, he hoped to eliminate doubts about the Axiom of Choice, and the meaningfulness of the study of the infinite, by showing that no proof could lead to a contradiction, thus showing mathematics to be consistent. He also hoped to show that every well-formed mathematical statement could be either proved or refuted.

In 1930, a young Austrian mathematician named Kurt Gödel took the first step on this project by showing that the accepted formulation of the notion of rigorous proof was in fact complete. However, in the next two years, he went on to show that Hilbert's program was doomed, because no finitely describable set of axioms suffices to decide all the statements even in finitary arithmetic. In addition, no such system can be shown to be consistent, except by making even stronger assumptions.

I will prove Gödel's First Incompleteness Theorem, and along the way give suggestions about how to see that the Completeness Theorem, and the Second Incompleteness Theorem are also true. I'll also aim to show some ways in which these theorems are and aren't relevant for ordinary mathematics.

*I am the very model of a modern Major General,
I've information vegetable, animal, and mineral,
I know the kings of England, and I quote the fights historical
From Marathon to Waterloo, in order categorical;
I'm very well acquainted, too, with matters mathematical,
I understand equations, both the simple and quadratical,
About binomial theorem I'm teeming with a lot o' news,
With many cheerful facts about the square of the hypotenuse!*

- Gilbert & Sullivan $P \circ P$

The website for Many Cheerful Facts is
<http://www.math.berkeley.edu/~slofstra/mcf>