

# MANY CHEERFUL FACTS

presents

## A generalized theory of computation

a talk by Kevin Lin

12:10 – 1:00pm on Wednesday, April 11, in room 1015.

One definition of algorithm is a finite set of well-defined instructions for accomplishing some task which can be performed by a Turing-complete system, e.g. a digital computer with arbitrary memory. The problem with this definition of algorithm is that Turing machines are fundamentally discrete; the Turing model does not seem to adequately address algorithms which involve continuous things like real numbers, for example Newton's method for approximating roots of a differentiable function, or Gaussian elimination.

In 1989, Lenore Blum, Michael Shub, and Steve Smale introduced a simple generalization of the Turing machine: a machine over an arbitrary ring  $R$ , where the original Turing machine is recovered in the case  $R = \mathbb{Z}/2\mathbb{Z}$ . The BSS theory gives a formal framework in which we can study non-discrete algorithms and their properties. It allows us to pose and to answer questions such as: Is the set of "good" starting points for Newton's method (i.e. points that, in the limit, yield a root of the given function) decidable? Results about machines over the real or complex numbers may also provide new insights into important questions about discrete machines, such as the famous conjecture  $P \neq NP$ .

In this talk I will outline the basics of the BSS theory and discuss some of the aspects of the theory mentioned above.

*I am the very model of a modern Major General,  
I've information vegetable, animal, and mineral,  
I know the kings of England, and I quote the fights historical  
From Marathon to Waterloo, in order categorical;  
I'm very well acquainted, too, with matters mathematical,  
I understand equations, both the simple and quadratical,  
About binomial theorem I'm teeming with a lot o' news,  
With many cheerful facts about the square of the hypotenuse!*

- Gilbert & Sullivan  $P \circ P$