Solutions.

Practice Quiz

NO CALCULATORS ARE ALLOWED. JUSTIFY YOUR ANSWERS APPROPRIATELY.

1. Find $\frac{d}{dx}\int_{\ln x}^{\sin x} \arctan t \, dt$ $= \frac{d}{dx}\left(\int_{\ln x}^{1} \arctan t \, dt + \int_{1}^{\sin x} \arctan t \, dt \right)$

$$\frac{d}{dx}\left(\int_{1}^{\sin x} \arctan t \, dt \right) = \arctan(\sin x) \cdot \cos x \quad \text{(Chain Rule)}$$

$$\frac{d}{dx}\left(- \int_{1}^{\ln x} \arctan t \, dt \right) = -\arctan(\ln x) \cdot \frac{1}{x} \quad (\text{"})$$

soln : $\boxed{\arctan(\sin x) \cdot \cos x - \frac{\arctan(\ln x)}{x}}$

2. Evaluate $\int_0^1 \frac{2t + t^3}{1+t^4}\, dt$ $= \int_0^1 \frac{2t}{1+t^4}\, dt + \int_0^1 \frac{t^3}{1+t^4}\, dt$

$$\int_0^1 \frac{2t}{1+t^4}\, dt = \int_0^1 \frac{du}{1+u^2} = \arctan u \Big]_0^1 = \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

let $u = t^2$

$du = 2t\, dt$

$$\int_0^1 \frac{t^3}{1+t^4}\, dt = \int_1^2 \frac{1}{u} \cdot \frac{1}{4}\, du$$

$$= \frac{1}{4} \ln|u| \Big]_1^2 = \frac{1}{4}(\ln 2 - \underbrace{\cancel{\ln 1}}_{0})$$

[over]

$u = 1 + t^4$

$du = 4t^3\, dt$

$\frac{1}{4} du = t^3\, dt$

soln : $\boxed{\frac{\pi}{4} + \frac{\ln 2}{4}}$

Indet. form 0^0

3. Evaluate $\lim_{x\to 0^+} x^{\sin x} = y$

$$\ln y = \ln \lim_{x\to 0^+} x^{\sin x} = \lim_{x\to 0^+} \ln(x^{\sin x}) = \lim_{x\to 0^+} \sin x \cdot \ln x$$

"$0 \cdot -\infty$"

$$= \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{\sin x}} \quad \text{"}\frac{-\infty}{\infty}\text{"}$$

$$\overset{\text{L'Hospital's}}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-\cos x}{\sin^2 x}} = \lim_{x\to 0^+} \frac{\sin x}{x} \cdot \frac{-\sin x}{\cos x}$$

$$= \lim_{x\to 0^+} \underbrace{\frac{\sin x}{x}}_{1} \cdot \lim_{x\to 0^+} \underbrace{\frac{-\sin x}{\cos x}}_{0} = 0.$$

Soln: $e^{\ln y} = e^0$

$= \boxed{1}$

4. Evaluate $\lim_{x\to\infty} \frac{e^x + \sin x}{e^x + \sin x^2}$

divide by e^x top and bottom

$$= \lim_{x\to\infty} \frac{1 + \frac{\sin x}{e^x} \to 0}{1 + \frac{\sin x^2}{e^x} \to 0}$$

$= 1.$

$$\lim_{x\to\infty} \frac{\sin x}{e^x} = 0 = \lim_{x\to\infty} \frac{\sin(x^2)}{e^x}$$

because both $\sin x$ and $\sin x^2$ are bounded between -1 and 1

but $e^x \longrightarrow \infty$ as $x \longrightarrow \infty$.

(formalize with Squeeze Theorem)