Math 1B: Quiz 9

1. Solve the following differential equations
~~(a)~~ $t\ln t\frac{dr}{dt} + r = te^t$

normal form: $\frac{dr}{dt} + \frac{1}{t\ln t} r = \frac{e^t}{\ln t} \longrightarrow I(t) = e^{\int \frac{1}{t\ln t} dt} = e^{\ln(\ln t)} = \ln t$

$\int \frac{1}{t\ln t} dt = \ln(\ln t)$

so $\ln t \frac{dr}{dt} + \frac{1}{t} r = e^t$ is

solved by $\int \ln t \frac{dr}{dt} + \frac{1}{t} r \cdot dt = \int e^t \cdot dt$

$[\ln t \cdot r] = e^t + C \longrightarrow r(t) = \frac{e^t}{\ln t} + \frac{C}{\ln t}$

~~(b)~~ $\frac{dy}{dx} = x\sin 2x + y\tan x$, for $-\pi/2 < x < \pi/2$

normal form: $\frac{dy}{dx} - y\tan x = x\sin 2x \longrightarrow I(x) = e^{\int -\tan x\, dx} = e^{\ln(\cos x)} = \cos(x)$

$\int -\tan x\, dx = \ln(\cos x)$

So $\cos x \frac{dy}{dx} - y\sin x = x\sin 2x\cos x$

$\hookrightarrow$ (too long.)

3. Solve the initial-value problem
$2xy' + y = 6x$, where $y(4) = 20$

normal form: $y' + \frac{1}{2x} y = 3 \longrightarrow I(x) = e^{\int \frac{1}{2x} dx} = e^{\frac{1}{2}\ln(2x)} = e^{\ln((2x)^{1/2})} = \sqrt{2x}$

$\int \frac{1}{2x} dx = \frac{1}{2}\ln(2x)$

So: $\sqrt{2x}\, y' + \frac{1}{\sqrt{2}\sqrt{x}} y = 3\sqrt{2}\sqrt{x}$

Solve: $\int \sqrt{2x}\, y' + \frac{y}{\sqrt{2x}}\, dx = \int 3\sqrt{2}\sqrt{x}\, dx$

$[\sqrt{2x}\, y] = \frac{\cancel{3}\sqrt{2}\, x^{3/2}}{\cancel{3}/2} + C$

$y = 2x + \frac{C}{\sqrt{2x}}$

$y(4) = 2(4) + \frac{C}{\sqrt{8}} = 20$

$8\sqrt{8} + C = 20\sqrt{8}$

$C = \sqrt{8}(20-8)$

$= 12\sqrt{8}$

$= 24\sqrt{2}$.

ANS: $y = 2x + \frac{24}{\sqrt{x}}$