Answers.

Math 1B: Quiz 12

THERE ARE TWO SIDES TO THIS QUZ. BE SURE TO SHOW ALL YOUR WORK.

1. Solve the recursion relation

$$c_{n+1} = \frac{c_{n-1}}{n+1}$$

for $n \geq 1$ if you know that $c_1 = 0$ and c_0 is arbitrary.

This relation is $c_{n+2} = \frac{c_n}{n+2}$; $n \geq 0$. So

$\underline{n=0}$: $c_2 = \frac{c_0}{2}$

$\underline{n=1}$ $c_3 = \frac{c_1}{3} = 0$

$\underline{n=2}$: $c_4 = \frac{c_2}{4} = \frac{c_0}{4 \cdot 2}$

$\underline{n=3}$. $c_5 = \frac{c_3}{5} = 0$

$\underline{n=4}$ $c_6 = \frac{c_4}{6} = \frac{c_0}{6 \cdot 4 \cdot 2}$

For $m = 2k$, $k = 3$

$$c_m = \frac{c_0}{6 \cdot 4 \cdot 2} = \frac{c_0}{(3 \cdot 2 \cdot 1)\, 2^3} = \frac{c_0}{k!\, 2^k}$$

For $m = 2k+1$, $c_m = 0$.

Thus
$$c_m = \begin{cases} \dfrac{c_0}{k!\, 2^k}, & m = 2k \\ 0, & m = 2k+1 \end{cases}$$

2. Use power series to solve the differential equation

$$y' = x^2 y$$

Assume $y = \sum_{n=0}^{\infty} c_n x^n$ is a solution.

$$y' = \sum_{n=1}^{\infty} n c_n x^{n-1} = \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n$$

$$y' - x^2 y = \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n - x^2 \sum_{n=0}^{\infty} c_n x^n$$

$$= \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n - \sum_{n=0}^{\infty} c_n x^{n+2}$$

$$= \sum_{n=0}^{\infty} (n+1) c_{n+1} x^n - \sum_{n=2}^{\infty} c_{n-2} x^n$$

$$= c_1 + 2c_2 x + \sum_{n=2}^{\infty} (n+1) c_{n+1} x^n - \sum_{n=2}^{\infty} c_{n-2} x^n$$

$$= \underbrace{c_1}_{=0} + \underbrace{2c_2}_{=0} x + \sum_{n=2}^{\infty} \underbrace{\left((n+1) c_{n+1} - c_{n-2}\right)}_{=0} x^n = 0$$

$$c_{n+1} = \frac{c_{n-2}}{n+1} \; ; \; n \geq 2 \quad : c_1 = 0 \quad : c_2 = 0.$$

$$c_{n+3} = \frac{c_n}{n+3} \; ; \; n \geq 0$$

$\underline{n=0}$: $c_3 = \frac{c_0}{3}$

$\underline{n=1}$: $c_4 = \frac{c_1}{4} = 0$

$\underline{n=2}$: $c_5 = \frac{c_2}{5} = 0.$

$\underline{n=3}$: $c_6 = \frac{c_3}{6} = \frac{c_0}{6 \cdot 3}$

$\vdots$

$\underline{n=6}$ $c_9 = \frac{c_6}{9} = \frac{c_0}{9 \cdot 6 \cdot 3}$

$$c_m = \frac{c_0}{9 \cdot 6 \cdot 3}$$

for $m = 3k = 9$
$\rightarrow k = 3.$

$$c_m = \frac{c_0}{(3 \cdot 2 \cdot 1)\, 3^3} = \frac{c_0}{k!\, 3^k}$$

$$c_m = \begin{cases} \frac{c_0}{k!\, 3^k} & ; \; m = 3k \\ 0 & ; \; m = 3k+1 \\ 0 & ; \; m = 3k+2 \end{cases}$$

So
$$y = \sum_{m=0}^{\infty} c_m x^m$$

$$= c_0 + \sum_{k=1}^{\infty} c_{3k} x^{3k}$$

$$= \boxed{c_0 + \sum_{k=1}^{\infty} \frac{c_0}{k!\, 3^k} x^{3k}}$$