notes

Worksheet 20 (Tuesday, 7/28):
These exercises are from section 11.3

Determine the sums of the following geometric series, when they are convergent:

1. (3.) $1 - \frac{1}{3^2} + \frac{1}{3^4} - \frac{1}{3^6} + \frac{1}{3^8} - \dots$

$a = 1$

$r = \frac{1/3^4}{1/3^2} = \frac{1}{9}$

$\text{sum} = \frac{1}{1-\frac{1}{9}} = \boxed{\frac{9}{8}}$

in-class

2. (6.) $3 + \frac{6}{5} + \frac{12}{25} + \frac{24}{125} + \frac{48}{625} + \dots$

3. (11.) $\frac{2}{5^4} - \frac{2^4}{5^5} + \frac{2^7}{5^6} - \frac{2^{10}}{5^7} + \frac{2^{13}}{5^8} + \dots$

4. (14.) $\frac{5^3}{3} - \frac{5^5}{3^4} + \frac{5^7}{3^7} - \frac{5^9}{3^{10}} + \frac{5^{11}}{3^{13}} + \dots$

$a = \frac{5^3}{3}$

$r = \frac{5^7/3^7}{-5^5/3^4} = \frac{-5^2}{3^3} = \frac{-25}{27}$

Sum an appropriate infinite series to find the rational number whose decimal expansion is given:

$\text{sum} = \frac{5^3/3}{1+25/27} = \frac{125}{\cancel{3}} \cdot \frac{\cancel{27}^{\,9}}{52} = \boxed{\frac{125(9)}{52}}$

in-class

5. (15.) $.272\overline{727}$

6. (29.) A patient receives 6 mg of a certain drug daily. Each day the body eliminates 30% of the amount of the drug present in the system. Estimate the total amount of the drug that should be present after extended treatment, immediately after a dose is given.

in-class

Determine the sums of the following series:

7. (36.) $\Sigma_{k=0}^{\infty} \frac{7}{10^k}$

in-class

8. (37.) $\Sigma_{j=1}^{\infty} 5^{-2j} = \frac{1}{5^2} + \frac{1}{5^4} + \frac{1}{5^6}$

$a = 1/25$

$r = \frac{1/5^6}{1/5^4} = \frac{1}{25}$

$\frac{1/25}{1 - 1/25} = \frac{1}{25} \cdot \frac{25}{24} = \boxed{\frac{1}{24}}$

9.* (42.) Show that the infinite series (in-class)

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots$$

diverges!

[Hint:
$\frac{1}{3}+\frac{1}{4}>\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$
$\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}>4(\frac{1}{8})=\frac{1}{2}$
etc..]

$s_n = a_1 + a_2 + \cdots + a_n$. Show $\lim_{n\to\infty} s_n = \infty$, then

series <u>diverges</u>!

$$s_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n}$$

$$> \underbrace{\left(\frac{1}{3}+\frac{1}{4}\right)}_{2} + \underbrace{\left(\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}\right)}_{4} + \underbrace{\left(\frac{1}{9}+\cdots+\frac{1}{16}\right)}_{8} + \underbrace{\left(\frac{1}{17}+\cdots\frac{1}{32}\right)}_{16 \text{ terms}} + \cdots \text{(some stuff)}$$

$$> 2\left(\frac{1}{4}\right) + 4\left(\frac{1}{8}\right) + 8\left(\frac{1}{16}\right) + 16\left(\frac{1}{32}\right) + \cdots + \left(\frac{2^{K-1}}{2^K}\right)$$ where $2^K < n$ is greatest possible

$$= \sigma(n)\cdot\frac{1}{2}$$ where $\sigma(n) = K \to \infty$

$\downarrow$

Thus $s_n > \sigma(n)\cdot\frac{1}{2} \to \infty$

& $s_n \to \infty$.