

argmin(p101):  $X = 1/2.5$   
 $Y = 23/2.5$

observation [1 0 1]    p101 =  $-1/27*x^2+1/18*x*y+1/27*y^2-1/54*x-5/54*y+1/6$

hidden [0 0 0]     $1/2*s00^2*t00*t01^2$      $1/27*x^2$   
 hidden [0 0 1]     $1/2*s00*s01*t00*t01*t11$      $2/27*x*(1-x)$   
 hidden [0 1 0]     $1/2*s01*s10*t01^2*t10$      $1/54*(1-x)*(1-y)$   
 hidden [0 1 1]     $1/2*s01*s11*t01*t10*t11$      $1/27*(1-x)*y$   
 hidden [1 0 0]     $1/2*s10*s00*t11*t00*t01$      $2/27*(1-y)*x$   
 hidden [1 0 1]     $1/2*s10*s01*t11^2*t00$      $4/27*(1-x)*(1-y)$   
 hidden [1 1 0]     $1/2*s11*s10*t10*t11*t01$      $1/27*y*(1-y)$   
 hidden [1 1 1]     $1/2*s11^2*t10*t11^2$      $2/27*y^2$

observation [1 1 0]    p110 =  $1/54*x^2-2/27*y^2-1/54*x+2/27*y+1/9$

hidden [0 0 0]     $1/2*s00^2*t00*t01^2$      $1/27*x^2$   
 hidden [0 0 1]     $1/2*s00*s01*t01^2*t10$      $1/54*x*(1-x)$   
 hidden [0 1 0]     $1/2*s01*s10*t00*t11*t01$      $2/27*(1-x)*(1-y)$   
 hidden [0 1 1]     $1/2*s01*s11*t01*t10*t11$      $1/27*(1-x)*y$   
 hidden [1 0 0]     $1/2*s10*s00*t11*t00*t01$      $2/27*(1-y)*x$   
 hidden [1 0 1]     $1/2*s10*s01*t10*t01*t11$      $1/27*(1-x)*(1-y)$   
 hidden [1 1 0]     $1/2*s11*s10*t11^2*t00$      $4/27*y*(1-y)$   
 hidden [1 1 1]     $1/2*s11^2*t10*t11^2$      $2/27*y^2$

observation [1 1 1]    p111 =  $-1/54*x^2+2/27*y^2-1/27*x+1/27*y+1/9$

hidden [0 0 0]     $1/2*s00^2*t01^3$      $1/54*x^2$   
 hidden [0 0 1]     $1/2*s00*s01*t01^2*t11$      $1/27*x*(1-x)$   
 hidden [0 1 0]     $1/2*s01*s10*t01^2*t11$      $1/27*(1-x)*(1-y)$   
 hidden [0 1 1]     $1/2*s01*s11*t01*t11^2$      $2/27*(1-x)*y$   
 hidden [1 0 0]     $1/2*s10*s00*t11*t01^2$      $1/27*(1-y)*x$   
 hidden [1 0 1]     $1/2*s10*s01*t11^2*t01$      $2/27*(1-x)*(1-y)$   
 hidden [1 1 0]     $1/2*s11*s10*t11^2*t01$      $2/27*y*(1-y)$   
 hidden [1 1 1]     $1/2*s11^2*t11^3$      $4/27*y^2$

! Analyze this model tropically!  
 How many inference functions are there?

This defines a surface in the 7-dimensional probability simplex. The prime ideal of this surface is generated by the six equations

$$\begin{aligned}
 p000 + p001 + p010 + p011 &= 1/2, \\
 p100 + p101 + p110 + p111 &= 1/2, \\
 4 p001 + 5 p010 &= 5 p101 + 4 p110 \\
 xx &= x * x \\
 xy &= x * y \\
 yy &= y * y
 \end{aligned}$$

where

$$\begin{aligned}
 xx &= 6 p010 - 12 p011 - 18 p101 - 12 p110 + 6 p111 + 4 \\
 xy &= -9 p010 - 18 p011 + 9 p101 + 18 p110 + 18 p111 - 2 \\
 yy &= -3/2 p010 - 6 p011 - 9/2 p101 - 6 p110 + 12 p111 + 1 \\
 x &= -12 p010 - 12 p011 + 6 p110 + 6 p111 + 2 \\
 y &= -6 p010 - 6 p011 + 12 p110 + 12 p111 - 1
 \end{aligned}$$

