3/19/12

A two-parameter Hidden Markov Model of length three on binary states:

Transition matrix: s00 = x s01 = 1-x
s10 = 1-y s11 = y

Output matrix: t00 = 2/3 t01 = 1/3
t10 = 1/3 t11 = 2/3

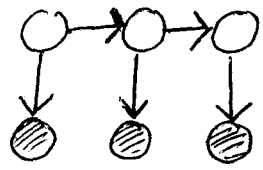

Figure 2 on page 12 of "The Mathematics of Phylogenomics"

observation [0 0 0]: p000 = 2/27*x^2-1/54*y^2+1/27*x-1/27*y+1/9

hidden [0 0 0] 1/2*s00^2*t00^3 4/27*x^2
hidden [0 0 1] 1/2*s00*s01*t00^2*t10 2/27*x*(1-x)
hidden [0 1 0] 1/2*s01*s10*t00^2*t10 2/27*(1-x)*(1-y)
hidden [0 1 1] 1/2*s01*s11*t00*t10^2 1/27*(1-x)*y
hidden [1 0 0] 1/2*s10*s00*t10*t00^2 2/27*(1-y)*x
hidden [1 0 1] 1/2*s10*s01*t10^2*t00 1/27*(1-x)*(1-y)
hidden [1 1 0] 1/2*s11*s10*t10^2*t00 1/27*y*(1-y)
hidden [1 1 1] 1/2*s11^2*t10^3 1/54*y^2

observation [0 0 1] p001 = -2/27*x^2+1/54*y^2+2/27*x-1/54*y+1/9

hidden [0 0 0] 1/2*s00^2*t00^2*t01 2/27*x^2
hidden [0 0 1] 1/2*s00*s01*t00^2*t11 4/27*x*(1-x)
hidden [0 1 0] 1/2*s01*s10*t00*t10*t01 1/27*(1-x)*(1-y)
hidden [0 1 1] 1/2*s01*s11*t00*t10*t11 2/27*(1-x)*y
hidden [1 0 0] 1/2*s10*s00*t10*t00*t01 1/27*(1-y)*x
hidden [1 0 1] 1/2*s10*s01*t10*t00*t11 2/27*(1-x)*(1-y)
hidden [1 1 0] 1/2*s11*s10*t10^2*t01 1/54*y*(1-y)
hidden [1 1 1] 1/2*s11^2*t10^2*t11 1/27*y^2

observation [0 1 0] p010 = 1/27*x^2+1/18*x*y-1/27*y^2-5/54*x-1/54*y+1/6

hidden [0 0 0] 1/2*s00^2*t00^2*t01 2/27*x^2
hidden [0 0 1] 1/2*s00*s01*t00*t01*t10 1/27*x*(1-x)
hidden [0 1 0] 1/2*s01*s10*t00^2*t11 4/27*(1-x)*(1-y)
hidden [0 1 1] 1/2*s01*s11*t00*t10*t11 2/27*(1-x)*y
hidden [1 0 0] 1/2*s10*s00*t10*t00*t01 1/27*(1-y)*x
hidden [1 0 1] 1/2*s10*s01*t10^2*t01 1/54*(1-x)*(1-y)
hidden [1 1 0] 1/2*s11*s10*t10*t11*t00 2/27*y*(1-y)
hidden [1 1 1] 1/2*s11^2*t10^2*t11 1/27*y^2

observation [0 1 1] p011 = -1/27*x^2-1/18*x*y+1/27*y^2-1/54*x+2/27*y+1/9

hidden [0 0 0] 1/2*s00^2*t00*t01^2 1/27*x^2
hidden [0 0 1] 1/2*s00*s01*t00*t01*t11 2/27*x*(1-x)
hidden [0 1 0] 1/2*s01*s10*t00*t11*t01 2/27*(1-x)*(1-y)
hidden [0 1 1] 1/2*s01*s11*t00*t11^2 4/27*(1-x)*y
hidden [1 0 0] 1/2*s10*s00*t10*t01^2 1/54*(1-y)*x
hidden [1 0 1] 1/2*s10*s01*t10*t01*t11 1/27*(1-x)*(1-y)
hidden [1 1 0] 1/2*s11*s10*t10*t11*t01 1/27*y*(1-y)
hidden [1 1 1] 1/2*s11^2*t10*t11^2 2/27*y^2

observation [1 0 0] p100 = 1/27*x^2-1/18*x*y-1/27*y^2+2/27*x-1/54*y+1/9

hidden [0 0 0] 1/2*s00^2*t00^2*t01 2/27*x^2
hidden [0 0 1] 1/2*s00*s01*t00*t01*t10 1/27*x*(1-x)
hidden [0 1 0] 1/2*s01*s10*t00*t10*t01 1/27*(1-x)*(1-y)
hidden [0 1 1] 1/2*s01*s11*t01*t10^2 1/54*(1-x)*y
hidden [1 0 0] 1/2*s10*s00*t11*t00^2 4/27*(1-y)*x
hidden [1 0 1] 1/2*s10*s01*t10*t00*t11 2/27*(1-x)*(1-y)
hidden [1 1 0] 1/2*s11*s10*t10*t11*t00 2/27*y*(1-y)
hidden [1 1 1] 1/2*s11^2*t10^2*t11 1/27*y^2

Does the likelihood function have local maxima?
Can you write the MLE analytically?
How does the EM algorithm perform?

ML degree = 53?