Problem Set 9 (due November 12)
MATH 110: Linear Algebra

Each problem is worth 10 points.

PART 1

1. Curtis **p. 161** 10.
2. Curtis **p. 226** 7.
3. Curtis **p. 243** 6.

PART 2

Problem 1(20)

Let V be a real vector space of functions spanned by the set of real values functions $\{e^x, xe^x, x^2e^x, e^{2x}\}$ and let T be the linear transformation $T : V \to V$ defined by $T(f) = f'$, the derivative of f. Find the Jordan canonical form of T.

Problem 2(10)

Prove that if V is isomorphic to W then V^* is isomorphic to W^*. Is the converse true (prove or give a counterexample)?

Problem 3 (10)

a) Let $T : R \to R$ be a linear transformation. Show that $T(x) = cx$ where $c \in R$ is some constant.

b) Let $T : R_2 \to R$ be a linear transformation. Show that $T(x, y) = c_1x + c_2y$.

c) Generalize parts a) and b) to a linear transformation of the form $T : R_n \to R$.

d) Show that every plane through the origin in R_3 may be identified with the null space of an element in $(R_3)^*$.

Problem 4 (10)

Let $T : M_{n,n}(R) \to M_{n,n}(R)$ be a linear transformation from the vector space of $n \times n$ matrices over R into itself, where $T(A) = A^t$. Find the minimal polynomial of T.

PART 3 - Optional Problem

Prove the lights and switches result combinatorially (i.e., without using linear algebra).