Problem Set 1 (due Friday September 3)
MATH 110: Linear Algebra

PART 1

The following problems are each worth 5 points..

1. Curtis **p. 14** 1.
2. Curtis **p. 15** 4.
3. Curtis **p. 25** 3.
4. Curtis **p. 33** 3.
5. Curtis **p. 37** 4.

PART 2

Remember that the starred problem is non collaborative.

Problem 1 (5)

Let $V = R^+$, the set of positive real numbers. Define the "sum" of two elements x, y in V to be their product xy (in the usual sense), and define "multiplication of an element x in V by a scalar c to be x^c. Prove that V is a real linear space with 1 as the zero element.

Problem 2 (10)

Let V be the vector space of all real valued functions defined on the real line ($F(R)$). Consider the n exponential functions:

$$u_1(x) = e^{a_1 x}, \ldots, u_n(x) = e^{a_n x}$$

where $a_1, a_2, \ldots, a_n$ are distinct real numbers. Show that these n functions are independent.

Problem 3*(5)

Curtis **p. 26** 10.

In order to do this problem you will have to read some definitions and theorems in Curtis (3.7, 3.9, 3.10 and 3.11).

Problem 4 (15)

Let V be a finite dimensional vector space and let S be a subspace of V. Prove each of the following statements:

a) S is finite dimensional and $dim\ S \leq dim\ V$.
b) $dim\ S = dim\ V$ if and only if $S = V$.
c) Every basis for S is part of a basis for V.
d) A basis for V need not contain a basis for S.
e) Is the union of two subspaces always a subspace? Explain.

PART 3 - Optional Problems

1. Recall that the **cross product** of two vectors A and B in R_3 is defined by

$$A \times B =< a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1 >$$

where $A =< a_1, a_2, a_3 >$ and $B =< b_1, b_2, b_3 >$.

Let $\mathbf{i}, \mathbf{j}, \mathbf{k}$ be the unit coordinate vectors in R_3. Notice that the cross product is not associative. For example,

$$\mathbf{i} \times (\mathbf{i} \times \mathbf{j}) = \mathbf{i} \times \mathbf{k} = -\mathbf{j} \ \ \textit{but} \ \ (\mathbf{i} \times \mathbf{i}) \times \mathbf{j} = 0 \times \mathbf{j} = 0.$$

Thus, given k vectors $v_1, v_2, \ldots, v_k$ in R_3, in order to make the expression

$$v_1 \times v_2 \times \cdots \times v_k$$

well defined, it is necessary to insert parentheses to indicate the order of evaluation. We will define an **association** to be an insertion of $k-2$ parentheses so that the order of evaluation is determined. For example,

$$(v_1 \times v_2) \times (v_3 \times v_4) \ \textit{and} \ ((v_1 \times v_2) \times v_3) \times v_4$$

are two different associations.

Prove that if two associations of $v_1 \times v_2 \times \cdots \times v_k$ are given, there exists an assignment of $\mathbf{i}, \mathbf{j}, \mathbf{k}$ to $v_1, v_2, \ldots, v_k$ such that when evaluated with the assignment, the two associations are equal and nonzero (this means that you cannot set $v_1 = v_2 = \cdots = v_k$).