Reasoning and Formal Modelling for Forensic Science Lecture 3

Prof. Dr. Benedikt Löwe

2nd Semester 2010/11

Binary connectives from last week.

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p	true	true	false	false
q	true	false	true	false
$p \vee q$	true	true	true	false
$p \text{ XOR } q$	false	true	true	false
$p \rightarrow q$	true	false	true	true
$p \leftrightarrow q$	true	false	false	true
$p \wedge q$	true	false	false	false

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$\vee$ disjunction / inclusive or. $\rightarrow$ implication. $\leftrightarrow$ equivalence.
$\wedge$ conjunction.

A notation convention (1).

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Writing

p	`true`	`true`	`false`	`false`
q	`true`	`false`	`true`	`false`
$p \vee q$	`true`	`true`	`true`	`false`

is rather tedious.

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We can shorten it to

$\vee$	T	F
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F	T	F.

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Note that now we need to specify whether we read it "by rows" or "by columns".

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is rather tedious.

We can shorten it to

$\vee$	T	F
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Note that now we need to specify whether we read it "by rows" or "by columns". For instance, the upper right entry in the matrix stands for row T and column F: but does this mean "p is true and q is false" or "q is true and p is false"?

A notation convention (2).

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To make this more interesting:

$\rightarrow$	T	F
T	?	?
F	?	?.

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One of the remaining entries stands for "false implies true" (which should be true) and the other one for "true implies false" (which should be false). But is row T, column F "false implies true" or rather column T, row F?

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Convention 1. In our matrices, the **rows** stand for the first entry of the binary connective, and the **columns** for the second:

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$\rightarrow$	T	F
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Our binary connectives according to **Convention 1**.

$\vee$	T	F
T	T	T
F	T	F

XOR	T	F
T	F	T
F	T	F

$\rightarrow$	T	F
T	T	F
F	T	T

$\leftrightarrow$	T	F
T	F	F
F	F	T

$\wedge$	T	F
T	T	F
F	F	F.

An observation: relating unary and binary connectives.

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In the table

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we now see several unary truth tables:

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Each of the rows and each of the columns corresponds to a unary truth table.

You are asked to make this observation more precise in **Homework Exercise B**.

A second convention.

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We can simplify

$\to$	T	F
T	T	F
F	T	T

even further:

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$\rightarrow$	T	F
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even further:

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Convention 2. In the above diagram, the left column stands for "true", the right column stands for "false". The upper row stands for "true", the lower row for "false".

Using truth tables for reasoning (1).

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Modus ponens

From Wikipedia, the free encyclopedia

In classical logic, **modus ponendo ponens** (Latin for *the way that affirms by affirming*;[1] often abbreviated to **MP** or **modus ponens**) is a valid, simple argument form sometimes referred to as **affirming the antecedent** or **the law of detachment**. It is closely related to another valid form of argument, *modus tollens*.

Modus ponens is a very common rule of inference, and takes the following form:

> If *P*, then *Q*.
> *P*.
> Therefore, *Q*.[2]

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$$((p \to q) \wedge p) \to q.$$

Using truth tables for reasoning (2).

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We say that a formula is valid if, no matter which truth values I plug in for the propositional variables, the truth tables for the connectives calculate the value of the entire formula as true.

Using truth tables for reasoning (2).

$$((p \to q) \land p) \to q.$$

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$((p \to q) \land p) \to q$ consists of subformulas:

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$((p \to q) \wedge p) \to q$ consists of subformulas:

- $p \to q$ and
- $(p \to q) \wedge p$.

We need two truth tables: the one for $\to$ and the one for $\wedge$:

T	F	T	F
T	T	F	F

Using truth tables for reasoning (3).

$$((p \rightarrow q) \land p) \rightarrow q$$

Relevant subformulas:				
	T	F	T	F
$p \rightarrow q$	T	T	F	F
$(p \rightarrow q) \land p$				

Using truth tables for reasoning (3).

$$((p \to q) \wedge p) \to q$$

Relevant subformulas:

$p \to q$	T	F	T	F
$(p \to q) \wedge p$	T	T	F	F

p	true	true	false	false
q	true	false	true	false

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$(p \rightarrow q) \wedge p$	true			

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T F T F

$p \to q$

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> *A: "If we find his fingerprints on the knife, he is the murderer."*
> *A & B process the knife and find matching fingerprints.*
> *B: "But I still don't believe that he is the murderer: maybe his fingerprints were already on the knife before the night of the murder."*
> *A: "You might be right."*

Equivalences.

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Equivalences.

We say that two formulas φ and ψ are equivalent if the formula $\varphi \leftrightarrow \psi$ is valid.

Claim. The formulas $p \rightarrow q$ and $\neg q \rightarrow \neg p$ are equivalent.

In order to prove this, we need to prove that $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$ is valid. The relevant subformulas are $p \rightarrow q$, $\neg p$, $\neg q$, and $\neg q \rightarrow \neg p$.

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Let's check whether this is valid.

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The formula $(p \to q) \to (\neg p \to \neg q)$ is not valid!

Contraposition and “Converse Error” (3).

Correct rule (Contraposition): $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$.
Incorrect rule (“converse error”): $(p \rightarrow q) \rightarrow (\neg p \rightarrow \neg q)$.

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Because in natural language, we often assume the "equivalence reading" of "if ... then ...".

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Why is it so easy to make this error?

Because in natural language, we often assume the “equivalence reading” of “if ... then ...”. If you read “If p then q” as $p \leftrightarrow q$, then the prosecutors reasoning becomes:

$$(p \leftrightarrow q) \to (\neg p \leftrightarrow \neg q)$$

(which is valid, as you can check yourself).

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4. Calculate the values of all relevant subformulas recursively from top to bottom (since you listed the more complex formulas later, this will work).
5. The final row of your calculation will contain the formula from Step 1. It is valid if the row has only entries `true`. In all other cases, it is not valid.

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p	true	false
$\neg p$	false	true
$\neg\neg p$	true	false
$\neg\neg p \rightarrow p$	true	true

The rule of double negation (or duplex negatio) is valid.

The law of excluded middle.

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Every proposition is either true or false.

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The law of contradiction is valid.

De Morgan's Law.

Prof. Dr. Benedikt Löwe

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De Morgan's Law is valid.

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$p \wedge \neg p$	false	false	false	false
$(p \wedge \neg p) \rightarrow q$	true	true	true	true

A last law for today...

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$(p \to q) \lor (q \to p)$.

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Why does this valid law sound so weird?