

Reasoning and Formal Modelling for Forensic Science Lecture 3

Prof. Dr. Benedikt Löwe

2nd Semester 2010/11

Binary connectives from last week.

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p	true	true	false	false
q	true	false	true	false
$p \vee q$	true	true	true	false
$p \text{ XOR } q$	false	true	true	false
$p \rightarrow q$	true	false	true	true
$p \leftrightarrow q$	true	false	false	true
$p \wedge q$	true	false	false	false

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\vee disjunction / inclusive or. \rightarrow implication. \leftrightarrow equivalence.
 \wedge conjunction.

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Writing

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is rather tedious.

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We can shorten it to

\vee		T	F
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F		T	F

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Note that now we need to specify whether we read it “by rows” or “by columns”.

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is rather tedious.

We can shorten it to

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Note that now we need to specify whether we read it “by rows” or “by columns”. For instance, the upper right entry in the matrix stands for **row T** and **column F**: but does this mean “ p is true and q is false” or “ q is true and p is false”?

A notation convention (2).

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To make this more interesting:

\rightarrow	T	F
T	?	?
F	?	?.

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One of the remaining entries stands for “false implies true” (which should be true) and the other one for “true implies false” (which should be false). But is **row T, column F** “false implies true” or rather **column T, row F**?

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Convention 1. In our matrices, the **rows** stand for the first entry of the binary connective, and the **columns** for the second:

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\rightarrow	T	F
T	T	F
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Our binary connectives according to Convention 1.

\vee	T	F	XOR	T	F
T	T	T	T	F	T
F	T	F	F	T	F

\rightarrow	T	F	\leftrightarrow	T	F
T	T	F	T	F	F
F	T	T	F	F	T

\wedge	T	F
T	T	F
F	F	F

An observation: relating unary and binary connectives.

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In the table

\rightarrow	T	F
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You are asked to make this observation more precise in **Homework Exercise B**.

A second convention.

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\rightarrow	T	F
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even further:

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Convention 2. In the above diagram, the left column stands for “true”, the right column stands for “false”. The upper row stands for “true”, the lower row for “false”.

Using truth tables for reasoning (1).

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Wikipedia

Modus ponens

From Wikipedia, the free encyclopedia

In [classical logic](#), **modus ponendo ponens** ([Latin](#) for *the way that affirms by affirming*,^[1] often abbreviated to **MP** or **modus ponens**) is a [valid](#), simple [argument form](#) sometimes referred to as **affirming the antecedent** or **the law of detachment**. It is closely related to another valid form of argument, *modus tollens*.

Modus ponens is a very common [rule of inference](#), and takes the following form:

If *P*, then *Q*.

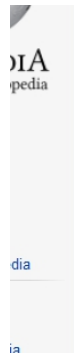
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Therefore, *Q*.^[2]

dia

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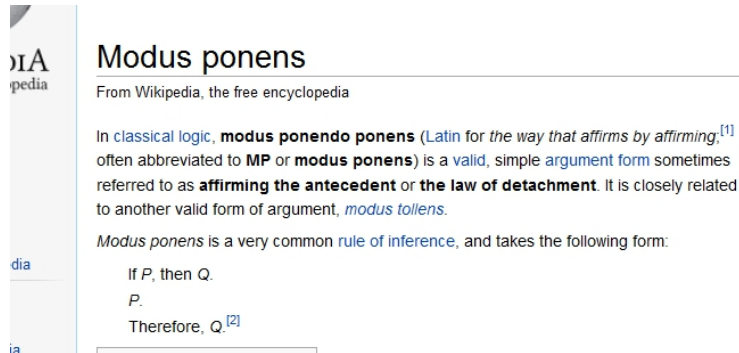
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Can we recover this rule in our system of truth tables?

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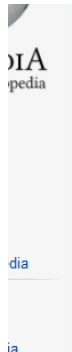
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First, we need to translate it into a formula:

$$((p \rightarrow q) \wedge p) \rightarrow q.$$

Using truth tables for reasoning (2).

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We say that a formula is **valid** if, no matter which truth values I plug in for the propositional variables, the truth tables for the connectives calculate the value of the entire formula as true.

Using truth tables for reasoning (2).

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We need two truth tables: the one for \rightarrow and the one for \wedge :

T	F	T	F
T	T	F	F

Using truth tables for reasoning (3).

$$((p \rightarrow q) \wedge p) \rightarrow q$$

Relevant subformulas:

$$p \rightarrow q$$

$$(p \rightarrow q) \wedge p$$

T

F

T

F

T

T

F

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Using truth tables for reasoning (3).

$$((p \rightarrow q) \wedge p) \rightarrow q$$

Relevant subformulas:

$p \rightarrow q$	T	F	T	F
$(p \rightarrow q) \wedge p$	T	T	F	F

p	true	true	false	false
q	true	false	true	false

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A: "If we find his fingerprints on the knife, he is the murderer."

A & B process the knife and find matching fingerprints.

B: "But I still don't believe that he is the murderer: maybe his fingerprints were already on the knife before the night of the murder."

A: "You might be right."

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<hr/>					
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Equivalences.

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Claim. The formulas $p \rightarrow q$ and $\neg q \rightarrow \neg p$ are equivalent.

In order to prove this, we need to prove that $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$ is valid. The relevant subformulas are $p \rightarrow q$, $\neg p$, $\neg q$, and $\neg q \rightarrow \neg p$.

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Let's check whether this is valid.

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The formula $(p \rightarrow q) \rightarrow (\neg p \rightarrow \neg q)$ is not valid!

Contraposition and “Converse Error” (3).

Correct rule (Contraposition): $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$.

Incorrect rule (“converse error”): $(p \rightarrow q) \rightarrow (\neg p \rightarrow \neg q)$.

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Why is it so easy to make this error?

Because in natural language, we often assume the “equivalence reading” of “if ... then ...”. If you read “If p then q ” as $p \leftrightarrow q$, then the prosecutors reasoning becomes:

$$(p \leftrightarrow q) \rightarrow (\neg p \leftrightarrow \neg q)$$

(which is valid, as you can check yourself).

The algorithm of proving by truth table.

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4. Calculate the values of all relevant subformulas recursively from top to bottom (since you listed the more complex formulas later, this will work).
5. The final row of your calculation will contain the formula from Step 1. It is valid if the row has only entries true. In all other cases, it is not valid.

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The rule of **double negation** (or **duplex negatio**) is valid.

The law of excluded middle.

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Every proposition is either true or false.

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The [law of excluded middle](#) (or [tertium non datur](#)) is valid.

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$p \wedge \neg p$	false	

The law of contradiction.

No proposition is both true and false.

Formalized as: $\neg(p \wedge \neg p)$.

This formula contains one propositional variable (p), and the subformulas $\neg p$ and $p \wedge \neg p$.

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The [law of contradiction](#) is valid.

De Morgan's Law.

Reasoning and
Formal Modelling
for Forensic
Science
Lecture 3

Prof. Dr. Benedikt
Löwe

De Morgan's Law.

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	$\neg p \vee \neg q$	false	true	true	true
	$\neg(p \wedge q)$	false	true	true	true
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De Morgan's Law is valid.

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$\neg p$	false	false	true	true
$p \wedge \neg p$	false	false	false	false
$(p \wedge \neg p) \rightarrow q$	true	true	true	true

A last law for today...

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Why does this valid law sound so weird?