Reasoning and Formal Modelling for Forensic Science Lecture 11

Prof. Dr. Benedikt Löwe

2nd Semester 2010/11

Partially Controlled Situation Sequences.

A partially controlled situation sequence consists of a finite number of *moments* t_1, ..., t_n, a fixed collection of individuals, properties and relations, and for each moment i, a partially controlled situation with relations S_i with these individuals, properties and relations.

The semantics at each given moment t_i is the usual semantics for partially controlled situations defining

$$\varphi \text{ is valid in } S_i$$

and

$$\varphi \text{ is invalid in } S_i.$$

Semantics of the temporal operators.

We fix a partially controlled situation sequence $S = (S_1, ..., S_n)$.

- $@_i\varphi$ is valid in S if φ is valid in S_i.
- $@_i\varphi$ is invalid in S if φ is not valid in S_i.
- $\mathrm{until}_i\varphi$ is valid in S if φ is valid in S_j for all $j = 1, ..., i$.
- $\mathrm{before}_i\varphi$ is valid in S if φ is valid in S_j for all $j = 1, ..., i-1$.
- $\mathrm{until}_i\varphi$ is invalid in S if φ is not valid in S_j for some $j = 1, ..., i$.
- $\mathrm{before}_i\varphi$ is invalid in S if φ is not valid in S_j for some $j = 1, ..., i-1$.
- $\mathrm{since}_i\varphi$ is valid in S if φ is valid in S_j for all $j = i, ..., n$.
- $\mathrm{after}_i\varphi$ is valid in S if φ is valid in S_j for all $j = i+1, ..., n$.
- $\mathrm{since}_i\varphi$ is invalid in S if φ is not valid in S_j for some $j = i, ..., n$.
- $\mathrm{after}_i\varphi$ is invalid in S if φ is not valid in S_j for some $j = i+1, ..., n$.

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The problem of *futura contingentia* (future contingents).

The Master argument.

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> *The Master argument seems to have been developed from the following starting points: There is a general conflict between the following three statements: (I) every past true proposition is necessary; and (II) the impossible does not follow from the possible; and (III) something is possible which neither is true nor will be true. Being aware of this conflict, Diodorus used the plausibility of the first two statements in order to show that (IV) nothing is possible that neither is nor will be true. (Epictetus, Dissertations 2.19.1)*

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- Therefore, p is not possible.

Ergo: Everything that is possible is true.

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In dealing with *futura contingentia*, we have to do with ontic uncertainty.

Different types of undecidedness

$\vee$	**T**	**?**	**F**
T	**T**	**T**	**T**
?	**T**	**?**	**?**
F	**T**	**?**	**F**

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But we would like to represent that we know that there will either be a sea-battle tomorrow or no sea-battle tomorrow.

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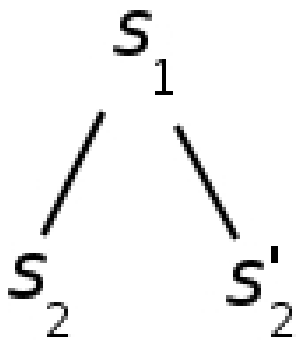

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Why trees?

Ontic uncertainty decreases over time: if you are looking into the past, you do not have ontic uncertainties anymore, only epistemic uncertainties. It is only with future contingents that you have ontic uncertainties.

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If we have a finite collection of moments t_0, ..., t_n in a sequence we can define "t_i lies in the future of t_j" if and only if $j < i$. Since sequences have no branching, they clearly have no branching into the past, so they are special cases of trees.

Trees of Partially Controlled Situations (1).

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A tree of partially controlled situations consists of a finite tree of *moments* t_1, ..., t_n together with its relation "lies in the future of", a fixed collection of individuals, properties and relations, and for each moment i, a partially controlled situation with relations S_i with these individuals, properties and relations.

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But we do not really have "its future", since there are many possible futures in a tree.

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If t_i is a moment, then it has several possible futures: each history that passes through t_i determines a possible future: if H is a branch that passes through t_i, we say that *the H-future of t_i* is the collection of moments t_j in H that lie in the future of t_i.

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Note: if H is a history in a tree of partially controlled situations, then it is in particular a sequence of partially controlled situations, and thus the semantics for sequences (Lecture 10) apply. So, if H is such a history, then "φ is valid in H" is defined as last time.

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We can now start to talk about possibility and introduce a new operator $\mathrm{possible}_i$ standing for "it is a possible future at moment t_i that".

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But this breaks that symmetry: there is no (meaningful) corresponding operator for "it is a possible past that".

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- $\mathrm{possible}_i\varphi$ is valid in S if there is a branch H passing through t_i such that φ is valid in H.
- $\mathrm{possible}_i\varphi$ is invalid in S if for all branches H passing through t_i, φ is not valid in H.

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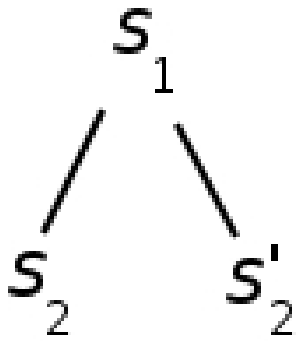
Once more the sea battle:



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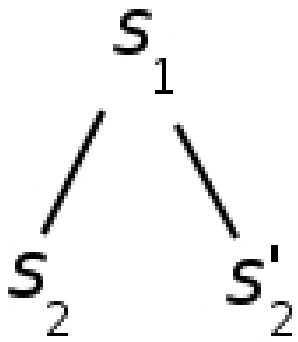

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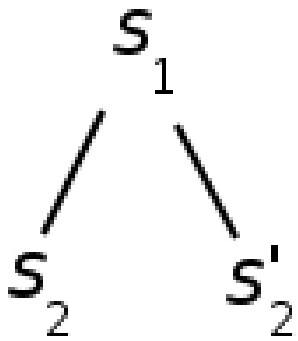

There are two histories: $H := (S_1, S_2)$ and $H' := (S_1, S'_2)$. Let us represent the sea-battle by a propositional letter p which is false in S_1 and S'_2 and true in S_2.

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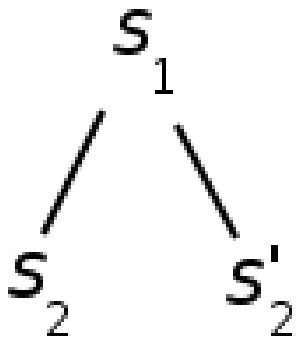

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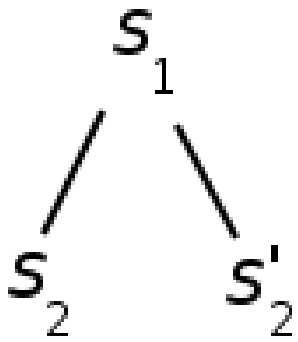

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In H, we have $\mathrm{possible}_i p$, whereas in H', we have $\mathrm{possible}_i \neg p$.

But we also get:

$$\mathrm{possible}_i \neg(p \vee \neg p) \text{ is invalid.}$$

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But sometimes, we are modelling things from the point of view of the agents in the story. It might be that we as modellers already know what happened later, but the agents in the story had to make their decisions under ontic uncertainty.

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Bart was travelling through France with his highly poisonous carpet viper. One evening, he stays in a fancy Sofitel, removes the snake from its terrarium and goes to bed together with his snake without any protection. The snake bites him a few minutes later. By sheer coincidence, the person in the room next to him is one of the leading experts on snake venom. He hears a suspicious sound, calls house keeping, finds Bart, administers the anti-venom that he carries with him, and saves Bart's life.

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In trying to figure out what happened (accident, suicide attempt, psychotic episode), we need to model the possible futures from Bart's perspective, even though we already know which of the future contingents happened.

An example.

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The police are investigating a series of burglaries. In each case, a well-to-do family has received a letter saying that they won a ticket to the opera, and when they went, the burglars broke into their home. The neighbourhood is on alert and police patrols are controlling the streets in the night. On 24 February, Philip Batch, one of the inspectors of the investigating team receives such a letter. The police plans to hide a number of agents in the house of the inspector, but Batch is uncertain: he thinks that this is a trap. By reducing the number of agents on the streets, the burglars would find it easier to get to their real target, which is still unknown to the police.