UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Core Logic

2007/2008; 1st Semester
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Homework Set # 7 *Deadline:* October 31st, 2007

Exercise 23 (8 points).

Consider the sentence *omnis philosophus praeter Socratem albus est* ("every philosopher except for Socrates is white").
Give a modern semantics for the *omnis praeter* construction: suppose we have a universe of discourse X, two predicates $\Phi, \Psi \subseteq X$ and $x \in X$. Give a formal definition such that

$$\mathbf{omnispraeter}(x, \Phi, \Psi)$$

is true if and only if *omnis* Φ *praeter* x *est* Ψ ("every Φ except for x is Ψ") (2 points).

Note. The "modern semantics" is not necessarily unique. There might be different semantics that describe the natural language sentences reasonably adequately.

Now consider the sophisma

($\star$) *omnis homo praeter Socratem excipitur*

("every man except for Socrates is excepted").

(1) Give a background story which describes a situation in which ($\star$) is true (2 points).
(2) Argue informally that ($\star$) is false (2 points).
(3) Solve the apparent contradiction by explaining the fallacy as a *secundum quid et simpliciter* (2 points).

Exercise 24 (4 points).

If X is any set and $\wp(X)$ is its power set (the set of all subsets of X), we call $\mathsf{Q} \subseteq \wp(X)$ a **generalized quantifier**. If $\Phi \subseteq X$ is a predicate on X, we say that $\mathsf{Q}\Phi$ holds (in words: "for Q-many x, $\Phi(x)$ holds") if $\Phi \in \mathsf{Q}$.

(1) Let $\forall := \{X\}$ and $\exists := \{A \subseteq X\,;\, A \neq \varnothing\}$. Argue that $\forall\Phi$ and $\exists\Phi$ have the intended meanings "for all x, $\Phi(x)$ holds" and "there is an x such that $\Phi(x)$ holds" (1 point each).
(2) Fix some $x \in X$ and give a definition of a generalized quantifier op_x that corresponds to the *omnis praeter* construction from **Exercise 23** (2 points).

Exercise 25 (6 points).

We are considering a new system of dialogic logic, called **strictly constructive**: we restrict the proponent in a way that he also can only react to the last move of the opponent and denote the corresponding semantic relation by $\models_{\text{sc}}$.

(1) Give a formal definition (in the style of the lecture, giving explicitly the rules for the two players) for $\models_{\text{sc}}$ (1 point).
(2) Find two different formulas φ such that $\models_{\text{sc}} \varphi$ and give dialogue proofs for them (1 point each).

(3) Find a formula φ such that $\models_{\text{dialog}} \varphi$ but not $\models_{\text{sc}} \varphi$. Give proofs of both claims (1½ point each).

Exercise 26 (4 points).

Give dialogue proofs of the following formulas in $\models_{\text{cl}}$ (1 point each):

- $\neg\neg\neg p \to \neg p$,
- $((p \to q) \wedge \neg q) \to \neg p$.

For both formulas, decide whether they are valid in $\models_{\text{dialog}}$ and give a dialogue argument for or against your claim (1 point each).