UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Core Logic

2007/2008; 1st Semester

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Homework Set # 5 *Deadline:* October 17th, 2007

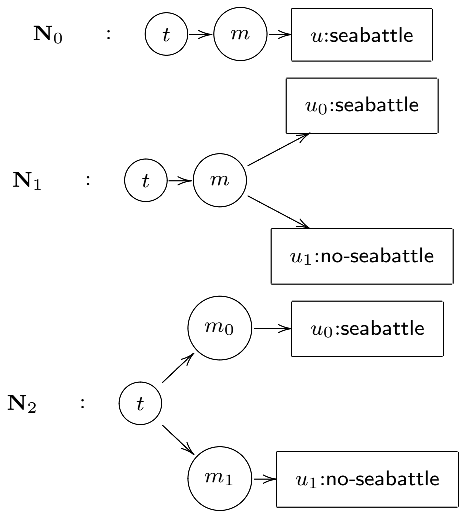
Exercise 16 (6 points).

A **naumachic model** is a quadruple $\langle M, U, \leq, S\rangle$ where M and U are finite non-empty sets, $\leq$ is a binary relation between M and U (***i.e.***, $\leq \ \subseteq M \times U$) and S is a function from U to $\{\mathsf{seabattle}, \mathsf{no\text{-}seabattle}\}$.

We call the elements of M **tomorrows**, the elements of U **DATs** (for "**D**ay **A**fter **T**omorrow"), if $m \leq u$, we say that "u is a possible future of m", and if $S(u) = \mathsf{seabattle}$ we say that "there is a sea battle at u" (similarly, if $S(u) = \mathsf{no\text{-}seabattle}$ we say that "there is no sea battle at u"). Given a naumachic model $\mathbf{N} = \langle M, U, \leq, S\rangle$, we say

- $\mathbf{N} \models$ "There will be a sea battle the day after tomorrow" if for all $m \in M$ and all u such that $m \leq u$, $S(u) = \mathsf{seabattle}$.
- $\mathbf{N} \models$ "There will be no sea battle the day after tomorrow" if for all $m \in M$ and all u such that $m \leq u$, $S(u) = \mathsf{no\text{-}seabattle}$.
- $\mathbf{N} \models$ "Tomorrow it will be determined whether there is a sea battle the day after tomorrow" if for all $m \in M$ the following holds: all u such that $m \leq u$ have the same value of $S(u)$.

We consider the following four pictures that represent naumachic models (the node t stands for "today", not represented in the formal model; the m_i are the tomorrows; the u_i are the DATs, the arrows indicate the $\leq$ relation, and u_i:seabattle means $S(u_i) = \mathsf{seabattle}$).



Are the following statements true or false (1 point each)?

(1) In $\mathbf{N}_0$, there will be a sea battle the day after tomorrow.
(2) In $\mathbf{N}_1$, there will be a sea battle the day after tomorrow.

(3) In $\mathbf{N}_2$, there will be a sea battle the day after tomorrow.
(4) In $\mathbf{N}_0$, it will be determined tomorrow whether there is a sea battle the day after tomorrow.
(5) In $\mathbf{N}_1$, it will be determined tomorrow whether there is a sea battle the day after tomorrow.
(6) In $\mathbf{N}_2$, it will be determined tomorrow whether there is a sea battle the day after tomorrow.

Exercise 17 (10 points).

Returning to the sheep of **Exercise 7** and **Exercise 9** and using the ideas of a naumachic model from **Exercise 16**, develop a semantics for sheep, owners and birth that allows us to talk about future contingents (4 points; be formally precise about your definitions). Your model should allow the construction of models of the formalizations of the following sentences and their negations:

- For some shepherd, it is not yet determined whether all of his sheep will give birth tomorrow.
- Tomorrow it will be determined whether all shepherds have a sheep that will give birth the day after tomorrow.

For both sentences, give models that make the sentence true and false and formally show that they do (1½ point for each of the four models; six points in total).

Exercise 18 (5 points).

Read the paper

> Christopher J. **Martin**, The Logic of Negation in Boethius, **Phronesis** 36 (1991), p. 277–304

(you can find a link to the PDF file on the webpage) and answer the following questions briefly:

- Boethius claims that "among the Peripatetics only Theophrastus and Eudemus made even the barest beginnings" of a theory of hypothetical syllogisms. Explain (in at most three sentences) why, according to Martin, material found in Avicenna casts some doubt on this claim. (p. 295; 3 points).
- McCall calls the propositional principle $(p \to q) \to \neg(p \to \neg q)$ "Boethius' principle". Martin disagrees. If Martin were to call this "X's principle", who would be X (1 point)?
- Martin claims that propositional logic was invented three times in western civilization? Who were these three inventors (1 point)?