Math 1a section worksheet, 12/9/04

1. Consider the solid generated by rotating the region bounded by $y = 2x^2$ and $y = x^2 + 4$ around the y axis. Compute its volume using

   (a) horizontal cross-sections perpendicular to the y-axis

   (b) cylindrical shells

2. Suppose the earth is a perfect solid sphere of radius r. We cut it using a horizontal plane halfway between the North Pole and the Equators. (In other words, the plane is the perpendicular bisector of the segment between the North Pole and the center.) Then we remove the part containing the North Pole. Set up an integral for the volume of the remaining piece.

3. Consider the solid generated by rotating the region bounded by $y = x$ and $y = \sqrt{x}$ around the y axis. We already computed its volume. Now compute its volume using cylindrical shells and check that we get the same answer.

4. Consider the solid generated by rotating the region bounded by $y = x^2$, $y = 0$, and $x = 3$ around the line $y = 4$. Compute its volume.

5. (This problem is about earlier topics of the semester.) We computed the volume of the solid donut in exercise 6.2.61. It can be shown that the surface area is $4\pi^2 rR$. When a donut shop makes donuts, it applies chocolate glaze on the entire surface.

   (a) Suppose the glaze has thickness dr. Approximate the volume of the glaze. (Hint: After we apply the glaze, the donut has radii R and $r + dr$ instead.)

   (b) On each donut, the cost of glaze per unit^2 of surface area is a constant G, while the cost of dough per unit^3 is a constant D. The shop charges $P(R + r)$ per donut for some constant P, to make the price proportional to a perceived "size." Assume that $P > 4\pi^2 RG$. Assume that R is a constant but that r is allowed to vary. What is the range of allowed values for r? What value of r maximizes the profit per donut? (There will be two different kinds of answers for the best r, depending on relative values of R, G, D, P.)

   (c) Open a donut shop.