

Stereographic projection:

$$\begin{aligned}x &= X/(1 - Z) & X &= 2x/(|z|^2 + 1) \\y &= Y/(1 - Z) & Y &= 2y/(|z|^2 + 1) \\& & Z &= (|z|^2 - 1)/(|z|^2 + 1).\end{aligned}$$

Cauchy-Riemann equations:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}.$$

Harmonic conjugate:

$$\begin{aligned}v(x, y) &= \int_{y_0}^y \frac{\partial u}{\partial x}(x, t) dt - \int_{x_0}^x \frac{\partial u}{\partial y}(s, y_0) ds + C \\v(B) &= \int_A^B -\frac{\partial u}{\partial y} dx + \frac{\partial u}{\partial x} dy.\end{aligned}$$

Fractional linear transformation:

$$w = f(z) = \frac{z - z_0}{z - z_2} \frac{z_1 - z_2}{z_1 - z_0}.$$

Mean value property:

$$u(z_0) = \int_0^{2\pi} u(z_0 + re^{i\theta}) \frac{d\theta}{2\pi}.$$

Cauchy integral formula:

$$f^{(m)}(z) = \frac{m!}{2\pi i} \int_{\partial D} \frac{f(w)}{(w - z)^{m+1}} dw.$$

Power series and Laurent series:

$$a_k = \frac{1}{2\pi i} \oint_{|\zeta - z_0|=r} \frac{f(\zeta)}{(\zeta - z_0)^{k+1}} d\zeta.$$

Residue theorem:

$$\int_{\partial D} f(z) dz = 2\pi i \sum_{j=1}^m \text{Res}[f(z), z_j].$$

$$\text{Res}[f(z), z_0] = \lim_{z \rightarrow z_0} (z - z_0) f(z) \quad \text{Res}[f(z), z_0] = \lim_{z \rightarrow z_0} \frac{d}{dz} (z - z_0)^2 f(z) \quad \text{Res}\left[\frac{f(z)}{g(z)}, z_0\right] = \frac{f(z_0)}{g'(z_0)}$$

Argument principle:

$$\frac{1}{2\pi i} \int_{\partial D} \frac{f'(z)}{f(z)} dz = \frac{1}{2\pi} \int_{\partial D} d \arg(f(z)) = N_0 - N_\infty.$$

Winding number:

$$W(\gamma, z) = \frac{1}{2\pi i} \int_\gamma \frac{1}{w - z} dw = \frac{1}{2\pi} \int_\gamma d \arg z.$$

Inverse function theorem:

$$f^{-1}(w) = \frac{1}{2\pi i} \int_{|\zeta - z_0|=\rho} \frac{\zeta f'(\zeta)}{f(\zeta) - w} d\zeta, \quad |w - w_0| < \delta.$$