Stereographic projection:

$$x = X/(1-Z) \qquad X = 2x/(|z|^2+1)$$
$$y = Y/(1-Z) \qquad Y = 2y/(|z|^2+1)$$
$$Z = (|z|^2-1)/(|z|^2+1).$$

Cauchy-Riemann equations:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \qquad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}.$$

Harmonic conjugate:

$$v(x,y) = \int_{y_0}^{y} \frac{\partial u}{\partial x}(x,t)dt - \int_{x_0}^{x} \frac{\partial u}{\partial y}(s,y_0)ds + C$$
$$v(B) = \int_A^B -\frac{\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy.$$

Fractional linear transformation:

$$w = f(z) = \frac{z-z_0}{z-z_2}\frac{z_1-z_2}{z_1-z_0}.$$

Mean value property:

$$u(z_0) = \int_0^{2\pi} u(z_0 + re^{i\theta})\frac{d\theta}{2\pi}.$$

Cauchy integral formula:

$$f^{(m)}(z) = \frac{m!}{2\pi i}\int_{\partial D} \frac{f(w)}{(w-z)^{m+1}}dw.$$

Power series and Laurent series:

$$a_k = \frac{1}{2\pi i}\oint_{|\zeta-z_0|=r} \frac{f(\zeta)}{(\zeta-z_0)^{k+1}}d\zeta.$$

Residue theorem:

$$\int_{\partial D} f(z)dz = 2\pi i \sum_{j=1}^{m} \mathrm{Res}[f(z), z_j].$$

Argument principle:

$$\frac{1}{2\pi i}\int_{\partial D} \frac{f'(z)}{f(z)}dz = \frac{1}{2\pi}\int_{\partial D} d\arg(f(z)) = N_0 - N_\infty.$$

Inverse function theorem:

$$f^{-1}(w) = \frac{1}{2\pi i}\int_{|\zeta-z_0|=\rho} \frac{\zeta f'(\zeta)}{f(\zeta)-w}d\zeta, \qquad |w-w_0| < \delta.$$