Math 54: Linear Algebra & DEs	*Sept 23, 2009*	Sec 105: 11-12
Lecturer: Prof. Constantin Teleman	**Quiz 4**	*GSI: Koushik Pal*

Name:..

Problem 1. Suppose A is a square matrix such that A^4 is not invertible. Is it possible for A to be invertible? Explain. **(3 points)**

Solution. Since A^4 is not invertible, therefore $\det(A^4) = 0$. But, $\det(A^4) = (\det(A))^4$. Hence, $\det(A) = 0$. Thus, A is not invertible.

Problem 2. Show that if A is invertible, then adj A is invertible, and

$$(\text{adj } A)^{-1} = \frac{1}{\det A} A.$$

(2 points)

Solution. Suppose A is $n \times n$. Since A is invertible, A^{-1} exists and is given by

$$A^{-1} = \frac{1}{\det A} \text{ adj } A.$$

Therefore, adj $A = (\det A)A^{-1}$, and hence, $|\text{adj } A| = (\det A)^n.|A^{-1}| \neq 0$ (since both A and A^{-1} are invertible). Therefore, adj A is invertible, and its inverse is given by,

$$(\text{adj } A)^{-1} = \frac{1}{\det A} A.$$

Alternatively, starting from the fact that $A^{-1} = \frac{1}{|A|}$ adj A, one can also show that

$$\text{adj } A.\frac{1}{|A|} A = \frac{1}{|A|} A \text{ adj } A = I_n.$$

And hence, by definition, $(\text{adj } A)^{-1} = \frac{1}{|A|} A$.

Problem 3. Solve the following system of equations by Cramer's rule:

$$\begin{aligned} 3x + 4y + 5z &= 6 \\ 3x + 5y + 6z &= 7 \\ 4x + 6y + 8z &= 9 \end{aligned}$$

(5 points)

Solution. The solution to the above system is given by:

$$x = \frac{\begin{vmatrix} 6 & 4 & 5 \\ 7 & 5 & 6 \\ 9 & 6 & 8 \end{vmatrix}}{\begin{vmatrix} 3 & 4 & 5 \\ 3 & 5 & 6 \\ 4 & 6 & 8 \end{vmatrix}} = \frac{1}{2}, \quad y = \frac{\begin{vmatrix} 3 & 6 & 5 \\ 3 & 7 & 6 \\ 4 & 9 & 8 \end{vmatrix}}{\begin{vmatrix} 3 & 4 & 5 \\ 3 & 5 & 6 \\ 4 & 6 & 8 \end{vmatrix}} = \frac{1}{2}, \quad z = \frac{\begin{vmatrix} 3 & 4 & 6 \\ 3 & 5 & 7 \\ 4 & 6 & 9 \end{vmatrix}}{\begin{vmatrix} 3 & 4 & 5 \\ 3 & 5 & 6 \\ 4 & 6 & 8 \end{vmatrix}} = \frac{1}{2}$$