Additional errata for "Slope filtrations revisited" (12 Feb 13)

This list excludes corrections included in the published erratum. Thanks to Max Bender for reporting these and for suggestions concerning Lemma 2.6.3.

Lemma 2.5.3: in (a), $i \in \mathbb{Z}$ should be $i \geq 0$. In the last line of the proof of (a), both instances of n should be i. In the last line of the proof of (c), $n \to \infty$ should be $i \to \infty$.

Lemma 2.5.4: it should be assumed that $r > 0$.

Corollary 2.5.6: $v_{j,n}$ should be $v_{j,r}$.

Lemma 2.5.11: it should also be assumed that $r \in I$. The statement is also correct when $r \notin I$ provided that the right side of the inequality is finite, but this is not used anywhere.

Lemma 2.6.3: The proof as written contains several errors. A corrected and simplified proof runs as follows.

Let m be the height of x. Apply Proposition 2.5.5 to choose a semiunit presentation $\sum_{i=0}^{\infty} u_i \pi^i$ of x, and put

$$c = \min_{n \geq m} \{v_{n,r}(x - u_m \pi^m)\} - w_r(x).$$

Note that $c > 0$ by Lemma 2.5.3 and the fact that

$$v_{n,r}(u_i \pi^i) \geq w_r(x) + (1 - r/r_0) \qquad (i < m, n \geq m).$$

Define a sequence $\{y_l\}_{l=0}^{\infty}$ as follows. Put $y_0 = y$. Given y_l with $y_l - y$ divisible by x and $w_r(y_l) \geq w_r(y)$, if y_l has height less than m, we make take $z = y_l$ and be done with the proof of the lemma. So we may assume that y_l has height at least m, which means that $\min_n\{v_{n,r}(y_l)\}$ is achieved by at least one $n \geq m$. Choose a semiunit presentation $\sum_{i=0}^{\infty} u_{l,i}\pi^i$ of y_l, and put $y_l' = \sum_{i=m}^{N} u_{l,i}\pi^i$ for N chosen large enough so that $v_{n,r}(u_{l,i}\pi^i) \geq w_r(y_l) + c$ for $n \geq N$. As in the proof that $c > 0$, we have

$$v_{n,r}(y_l' - y_l) \geq w_r(y_l) + c \qquad (n \geq m).$$

Put

$$\begin{aligned} y_{l+1} &= y_l - y_l' \pi^{-m} x / u_m \\ &= (y_l - y_l') + y_l'(1 - \pi^{-m} x/u_m). \end{aligned}$$

Then for all $n \geq m$,

$$v_{n,r}(y_l'(1 - \pi^{-m}x/u_m)) \geq \min\{\min_{n' > n}\{v_{n,r}(y_l)\}, w_r(y_l) + c\},$$

so the same lower bound holds for $v_{n,r}(y_{l+1})$. This implies that $w_r(y_{l+1}) \geq w_r(y_l)$, so $w_r(y_{l+h}) \geq w_r(y_l)$ for all $h > 0$. We may assume that $y_{l+1}, y_{l+2}, \ldots$ also have height at least m. There must then exist h for which $w_r(y_{l+h}) \geq w_r(y_l) + c$: otherwise, the maximum index n for which $v_{n,r}(y_{l+h}) < w_r(y_l) + c$ decreases as h increases but is bounded below by m, contradiction.

It follows that the y_l converge to zero under w_r, and

$$y = x \sum_{l=0}^{\infty} (y_l - y_{l+1})/x \in \Gamma_r$$

is divisible by x, so we may take $z = 0$.

Remark 2.6.4: "[discreteness of the valuation] on K" should be "[...] on $\mathcal{O}$".

Lemma 2.6.7: The sentence starting "Moreover, if it is ever less than $\min_{n<0}\{v_{n,r'}(u_l x)\} + c$," should continue "then the smallest value of n for which $v_{n,r'}(u_{l+1}x) \leq \min_{n<0}\{v_{n,r'}(u_l x)\} + c$ is strictly greater than the smallest value of n for which $v_{n,r'}(u_l x) \leq \min_{n<0}\{v_{n,r'}(u_l x)\} + c$."

Proposition 2.6.8: the reference to Proposition 2.6.8 in the proof should be to Proposition 2.6.5.