Erratum for "Semistable reduction for overconvergent F-isocrystals, I: Unipotence and logarithmic extension"

Atsushi Shiho has pointed out an error in the proof of Proposition 3.5.3. The erroneous statements are the last two sentences of the proof: since the strict neighborhood V' may not have integral reduction, one cannot necessarily embed it isometrically into a field. A similar error occurs in Lemma 3.6.2, because X need not have integral reduction.

In the case of Lemma 3.6.2, this is most easily corrected by simply requiring X to have integral reduction, and making the same requirement in Proposition 3.6.9. This suffices because the only invocation of Lemma 3.6.2 is in Proposition 3.6.9, and all invocations of Proposition 3.6.9 occur in cases where X has integral reduction.

In the case of Proposition 3.5.3, the only invocation occurs in Lemma 5.1.1, in which we may assume $b > 0$. Under this extra hypothesis, we may correct the proof of Proposition 3.5.3 as follows. (Thanks to Shiho for feedback on this correction.)

Retain notation from the proof of Proposition 3.5.3. We have from the correct part of the proof that $H^0_{V_\lambda}(V_\lambda \times A^n_K[b,c], \mathcal{E}) \neq 0$. Take the $\mathcal{O}$-span of this set; it is a submodule of $\mathcal{E}$ stable under ∇. Restrict to $V^0_\lambda \times A^n_K[b,c]$, for V^0_λ the nonlogarithmic locus of V_λ; by Proposition 3.3.8, this restriction extends to a log-∇-submodule $\mathcal{F}$ of $\mathcal{E}$. On $(V^0_\lambda \cap]X[) \times A^n_K[b,c]$, $\mathcal{F}$ is constant relative to $V^0_\lambda \cap]X[$; we may infer the same conclusion on $]X[\times A^n_K[b,c]$ via Theorem 3.3.6 (since $\mathcal{E}$ is already known to be unipotent there).

In other words, the whole proof reduces to the case when $\mathcal{E}$ is constant, not just unipotent, on $]X[\times A^n_K[b,c]$. In this case, we must show that $\mathcal{E}$ is also constant on $V_\lambda \times A^n_K[b,c]$. We first prove that $g_\lambda : \pi_1^* H^0_{V_\lambda}(V_\lambda \times A^n_K[b,c], \mathcal{E}) \to \mathcal{E}$ is surjective for some λ. For $\mathbf{v} \in \Gamma(V \times A^n_K[d,e], \mathcal{E})$, the correct part of the proof of Proposition 3.5.3 shows that the sequence $D_l(\mathbf{v})$ converges on $V_\lambda \times A^n_K[b,c]$ for some λ. Moreover, as in Proposition 3.4.3, on $]X[\times A^n_K[b,c]$ we have the identity

$$\mathbf{v} = \sum_{J \in \mathbb{Z}^n} t_1^{j_1} \cdots t_n^{j_n} f(t_1^{-j_1} \cdots t_n^{-j_n})$$

(using the hypothesis that $b \neq 0$), so the cokernel of g_λ has no support on $]X[\times A^n_K[b,c]$. On $V_\lambda \times A^n_K[b,c]$, the support of this cokernel is a closed analytic subspace not meeting $]X[\times A^n_K[b,c]$; by the maximum modulus principle, it also fails to meet $V_{\lambda'} \times A^n_K[b,c]$ for some $\lambda' \in (1,\lambda]$. Hence for suitable λ, g_λ is surjective.

Over $V^0_\lambda \times A^n_K[b,c]$, the map g_λ is automatically a morphism in $\mathrm{LNM}_{V^0_\lambda \times A^n_K[b,c]}$ because there are no logarithmic singularities; hence Proposition 3.2.20 implies that the restriction of $\mathcal{E}$ to $V^0_\lambda \times A^n_K[b,c]$ is constant over V^0_λ.

To finish the proof that $\mathcal{E}$ is constant, it suffices to do so after extending scalars from K to a finite extension. By doing so, we may ensure that there exists a K-rational point $x \in A^n_K[b,c]$. Let $\mathcal{E}_0$ be the restriction of $\mathcal{E}$ to $V_\lambda \times \{x\}$, identified with V_λ. We then have an isomorphism of $\pi_1^*\mathcal{E}_0$ with $\mathcal{E}$ over $V^0_\lambda \times A^n_K[b,c]$ (because $\mathcal{E}$ is constant over V^0_λ); by Proposition 3.3.8 (applied to the graph of the isomorphism inside of $\pi_1^*\mathcal{E}_0 \oplus \mathcal{E}$), this extends to an isomorphism $\pi_1^*\mathcal{E}_0 \cong \mathcal{E}$ over $V_\lambda \times A^n_K[b,c]$.

We also note that Shiho has generalized the results of §3 in his new preprint "On logarithmic extension of overconvergent isocrystals" (arXiv:0806.4394), using somewhat different arguments. Hence one can also avoid the aforementioned errors simply by invoking Shiho's paper instead.