Math 1B - Midterm Review
April 5, 2008

1. Consider the area enclosed by graphs $y = 6x^{3/2}$, $y = 0$ and $x = 1$. Find
(a) the parameter of the region (i.e. sum of the length of its three "sides")
(b) the centroid of the region

2. A trough is 10 m long and its side has the shape of inverted equilateral triangle of length 1m. Suppose it is filled with water.
(a) what is the work required to pump water out at the top?
(b) What is the pressure on one of the triangular sides?
(c) what is the pressure on one of the rectantular sides?

Just to remind you that units can be confusing, repeat above if the dimensions were 10 ft and 1 ft?
mass density of water $\rho = 1000kg/m^3$
gravitational acceleration $g = 9.8m/s^2$
weight density of water $\delta = 62.5lb/ft^3$

3. A 30 kg object is suspended by a 10 m cable that weighs 1 kg/m. How much work is required to raise it by 3 m?

4. A 3.14 m spring with $k = 5N/m$ is already compressed 1m. How much work is required to compress it by 2 more meters?

5. A 10 L bathtub with 2 tabs is filled with 5 % alcohol solution. From one tab, 7 % alcohol solution is coming in at 1 L/min. From the other tab, 4 % alcohol solution is coming in at 0.5 L /min. Mixed solution overflows the tub. What is the percentage of alcohol after 10 minutes?

6. Solve the differential eqution $y' = xe^y$.

7. Consider sequence defined by $a_1 = 1$ and $a_{n+1} = \sqrt[3]{a_n + 6}$. Show that the sequence increase and is bounded above. What is the limit?

8. Use integral test to show that the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges and is less than 2. Sketch the graph to illustrate this.

9. Find limit, if exists, of the following sequences.
(a) $a_n = (1 - \frac{2}{n+1})^n$

(b) $a_n = \frac{n}{(\ln n)^3}$

(e) $a_n = (-1)^n \frac{\sin n}{n}$

10. Determine if the series converges. (some of these are from previous sessions)

(a) $\sum_{n=1}^{\infty} \frac{n^2}{n!}$

(b) $\sum_{n=1}^{\infty} (\frac{n+1}{n})^{3n} \cdot \frac{1}{3^n}$

(c) $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$

(d) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

(e) $\sum_{n=1}^{\infty} (\frac{\ln n}{n})^n$

(f) $\sum_{n=5}^{\infty} \frac{1}{\sqrt[3]{n-3}}$

(g) $\sum_{n=0}^{\infty} (1/4)^{n^2}$

(h) $\sum_{n=3}^{\infty} \frac{1}{(\ln n)^2}$

(i) $\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$

(j) $\sum_{n=0}^{\infty} (-1)^n \frac{n^2-1}{n^3-1}$

(k) $\sum_{n=1}^{\infty} \cos(\frac{1}{n^2}) \cos(n^2)$

(l) $\sum_{n=0}^{\infty} \frac{n^3 3^n}{n!}$

(m) $\sum_{n=1}^{\infty} \ln(\frac{n+1}{n})$

(n) $\sum_{n=1}^{\infty} \frac{2}{n^2+4n+3}$

11. Evaluate, if exists,

(a) $\sum_{n=3}^{\infty} \frac{2^(2n)}{(-5)^n}$

(b) $\sum_{n=3}^{\infty} \frac{1}{n^2-4}$