

MATH 54 (Ribet, GSI: Justin)

Quiz 9, April 11, 2007

Name: _____

PROBLEM ONE: THE GOOD [7 PTS.]

This problem deals with the differential equation $y'' - y = 0$.

(a) [2 pts.] Find a fundamental set of solutions. (Answer must be a set of functions!)

(b) [2 pts.] Solve for $y(t)$ if $y(0) = 2$, $y'(0) = 0$.

(c) [3 pts.] Rewrite the second-order equation as a system of two first-order linear equations. Translate the initial conditions of (b) as well.

PROBLEM TWO: THE BAD [1 PT.]

This problem deals with the differential equation $y'' + 2yy' = 0$.

It is *false* that a superposition (sum) of two solutions is a solution, that the Wronskian is given by Abel's formula, and that the initial conditions $y(0) = 1$, $y'(0) = 0$ lead to a unique solution. What is going on?

Answer: _____.

PROBLEM THREE: THE UGLY [5 PTS.]

This problem deals with the differential equation $(2 - t)(2 + t)y'' + 3(2 + t)y' + 4y = 0$.

(a) [1 pt.] Find an equivalent equation of the form $y'' + p(t)y' + q(t)y = 0$.

(b) [2 pts.] Given the initial conditions $y(0) = 1$, $y'(0) = 1$, determine the longest interval on which the equation must have a unique, twice-differentiable solution.

(c) [2 pts.] If $\{y_1, y_2\}$ is a fundamental set, and the Wronskian $W(y_1, y_2)$ is equal to 1 when $t = 0$, find a formula for $W(t)$.