



PROBLEM THREE: THE UGLY [5 PTS.]

This problem deals with the differential equation  $(2-t)(2+t)y'' + 3(2+t)y' + 4y = 0$ .

$$(2-t)(2+t)$$

- (a) [1 pt.] Find an equivalent equation of the form  $y'' + p(t)y' + q(t)y = 0$ .

$$y'' + \frac{3}{2-t}y' + \frac{4}{(2-t)(2+t)}y = 0$$

- (b) [2 pts.] Given the initial conditions  $y(0) = 1$ ,  $y'(0) = 1$ , determine the longest interval on which the equation must have a unique, twice-differentiable solution.

In answer to (a),  $P(t), Q(t)$  continuous except at  $t=2, t=-2$ . Solution must be defined at  $t=0$ ,

$$\therefore (-2, 2)$$

- (c) [2 pts.] If  $\{y_1, y_2\}$  is a fundamental set, and the Wronskian  $W(y_1, y_2)$  is equal to 1 when  $t = 0$ , find a formula for  $W(t)$ .

By Abel's theorem,  $W(t) = Ce^{-\int \frac{3}{2-t} dt}$

$$\int \frac{3}{2-t} dt = \int \frac{3}{u} \cdot -du = -3 \ln|2-t| + C \quad |2-t| = 2-t \text{ on } (-2, 2)$$

$$\Rightarrow W(t) = Ce^{3 \ln(2-t)} = (e^{\ln(2-t)})^3 = C(2-t)^3$$

When  $t=0$ :  $1 = W(t) = C \cdot 2^3 = 8C \Rightarrow C = \frac{1}{8} \Rightarrow W(t) = \frac{1}{8}(2-t)^3$