MATH 54 (Ribet), S205

Quiz 8, March 21, 2007 Name: ______________________

PROBLEM ONE [5 PTS.]

(Not based on a true story.)

The user `MegaGeniousD00d` on the Math 54 Google group has posted homework answers that differ from yours. The first problem asks you to find a diagonalization $A = S\Lambda S^{-1}$. The second problem asks you to find a diagonalization $A = Q\Lambda Q^T$, where Q is an orthogonal matrix.

Problem	Your	answer	MGD's	answer
5.3.22	$\Lambda = \begin{bmatrix} 0 & & \\ & 0 & \\ & & 1 \end{bmatrix}$	$S = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}$	$\Lambda = \begin{bmatrix} 0 & & \\ & 0 & \\ & & 1 \end{bmatrix}$	$S = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 4 & 0 \\ -4 & 8 & -1 \end{bmatrix}$
5.4.4	$\Lambda = \begin{bmatrix} -5 & \\ & 8 \end{bmatrix}$	$Q = \begin{bmatrix} \frac{3}{\sqrt{13}} & -\frac{2}{\sqrt{13}} \\ \frac{2}{\sqrt{13}} & \frac{3}{\sqrt{13}} \end{bmatrix}$	$\Lambda = \begin{bmatrix} -5 & \\ & 8 \end{bmatrix}$	$Q = \begin{bmatrix} 3 & -2 \\ 2 & 3 \end{bmatrix}$

(a) [2 pts.] In the first problem you're getting the same Λ but wildly different-looking S matrices. Explain how it's possible for both of your answers to be correct.

You and MGD could have (in fact, have) chosen different bases for the eigenspaces.

(b) [1 pt.] In the second problem, is MGD's Λ a diagonal matrix? Yes!

(c) [1 pt.] In the second problem, is MGD's Q an orthogonal matrix? NO

(d) [1 pt.] What would you tell MGD to do differently?

Orthogonal matrices must have orthonormal columns; if he doesn't normalize his eigenvectors, his answer won't work.

(Other advice was optional.)

PROBLEM TWO [5 PTS.]

Determine if the given matrix is diagonalizable. If so, find matrices S and Λ such that the given matrix equals $S\Lambda S^{-1}$.

$$A = \begin{bmatrix} 2 & 0 \\ 3 & -1 \end{bmatrix}$$

$\lambda I - A = \begin{bmatrix} \lambda-2 & 0 \\ -3 & \lambda+1 \end{bmatrix} \therefore \det(\lambda I - A) = (\lambda-2)(\lambda+1) \rightsquigarrow \lambda = 2, -1$

Distinct eigenvalues $\Rightarrow$ diagonalizable. (In general, diagonalizable if there exist n linearly independent eigenvectors.)

$\lambda = 2$: $\lambda I - A = \begin{bmatrix} 0 & 0 \\ -3 & 3 \end{bmatrix} \rightsquigarrow v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda = -1$: $\lambda I - A = \begin{bmatrix} -3 & 0 \\ -3 & 0 \end{bmatrix} \rightsquigarrow v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\boxed{\Lambda = \begin{bmatrix} 2 & \\ & -1 \end{bmatrix}, \quad S = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}}$$