MATH 54 (Ribet), S205

Quiz 6, March 7, 2007 Name: ______________________

PROBLEM ONE [5 PTS.]

(a) [2 pts.] Find the best-fit line $y = a_0 + a_1 x$ (in the least-squares sense) for the data points $(-1, 4)$, $(0, 7)$, and $(1, 16)$.

(b) [2 pts.] Same as (a), but with the middle data point repeated: i.e., find the least-squares solution to the (inconsistent) system

$$\begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 7 \\ 16 \end{bmatrix}$$

(c) [2 pts.] Calculate the total square error $(\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2)$ from (a)&(b).

(d) [1 pt.] Should the answers to (a) and (b) be the same? Why or why not?

PROBLEM TWO [4 PTS.]

The vectors v_1, v_2, v_3, and v_4 are given below. Let V be their span (a subspace of $\mathbb{R}^4$). Find $\dim V$ and an orthonormal basis for V.

$$v_1 = \begin{bmatrix} -1 \\ 0 \\ 2 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ 0 \\ -4 \\ -6 \end{bmatrix}, v_3 = \begin{bmatrix} -1 \\ -1 \\ 5 \\ 1 \end{bmatrix}, v_4 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 5 \end{bmatrix}.$$

Hint: You can still use the Gram-Schmidt process if the vectors are linearly dependent. If you obtain a zero vector, it's because the vector you started with was redundant, and you can just throw it out.