MATH 54 (Ribet), S205/S215

Quiz 3, February 7, 2007 Name: Norbert Wiener

PROBLEM ONE [4 PTS.]

The vectors $v_1 = \langle 3, 0, 4\rangle$ and $v_2 = \langle -1, 1, 7\rangle$ span a plane of $\mathbb{R}^3$. [Specifically, the plane $4x + 25y - 3z = 0$.] Use the Gram-Schmidt process to find an orthonormal coordinate system (i.e., a spanning set of perpendicular length-1 vectors) for this plane.

1. $e_1 \leftarrow \frac{v_1}{\|v_1\|}$: $e_1 = \boxed{\frac{1}{5}\langle 3,0,4\rangle}$

2. $e_2' \leftarrow v_2 - (v_2 \cdot e_1)e_1 = v_2 - \frac{v_2 \cdot v_1}{v_1 \cdot v_1} v_1$:

$$e_2' = \langle -1,1,7\rangle - \frac{-3+28}{25}\langle 3,0,4\rangle = \langle -4,1,3\rangle$$

2½. $e_2 \leftarrow \frac{e_2'}{\|e_2'\|}$: $e_2 = \boxed{\frac{1}{\sqrt{26}}\langle -4,1,3\rangle}$

PROBLEM TWO [3 PTS.]

Decide whether (a)–(c) are **sometimes**, **always**, or **never** true. [1 pt. each]:

(a) The projection of u on v is perpendicular to v (assuming that none of the vectors involved is zero).
(b) The span of a collection of vectors in $\mathbb{R}^n$ is a subspace of $\mathbb{R}^n$.
(c) The solution set of a linear system of equations in n variables is a subspace of $\mathbb{R}^n$.

(a) Never (it's Parallel)

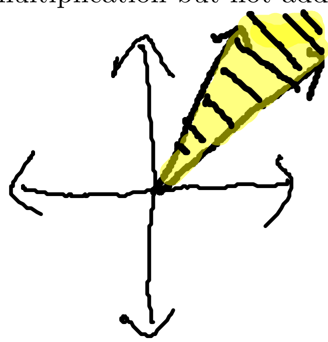
(b) Always (that's the whole point!)

(c) Sometimes:
* If it's $Ax=0$, get $NS(A)$, which is a subspace.
* If it's $Ax=b, b\neq 0$, the solution set isn't closed under, well, anything. (or it's $\emptyset$.)

PROBLEM THREE [3 pts.]

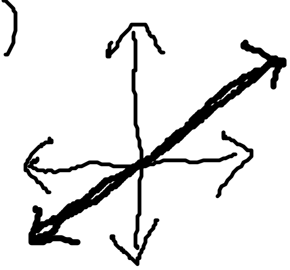
Three subsets of $\mathbb{R}^2$ are drawn below. Set (a) is a subspace of $\mathbb{R}^2$. Set (b) is closed under addition but not scalar multiplication. Set (c) is closed under scalar multiplication but not addition. Identify the three sets.

(i)



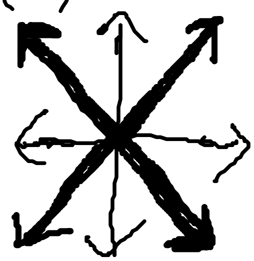
b

(ii)



a

(iii)



c