

MATH 54 S215/S205: REVIEW FOR HILL §3.7–END

Note: It may be a good idea to look back at my review problems for the MT2 review for coordinate vectors and transition matrix material!

- (1) The continuous 2π -periodic functions form a vector space. (Why?) Show that $A_0(f) = a_0$, $A_n(f) = a_n$, and $B_n(f) = b_n$ (a_0 , a_n , and b_n being the usual Fourier coefficients for f) are linear transformations. Can any of them be represented by a matrix? Why or why not?
- (2) Explain why $f \cdot g = \int_0^1 f(x)g(x)dx$ is an inner product on $C[0, 1]$ but not on $C[-1, 1]$.
- (3) In a vector space with inner product, prove that the vectors orthogonal to a given vector x form a subspace. In the case of \mathbb{R}^n , prove this subspace is $NS(x^T)$.
- (4) If Q is an orthogonal matrix with entries $\pm 1/4$, what are the dimensions of Q ?
- (5) A is a 2×2 matrix with $\text{Tr}(A) = 3$ and $\det(A) = 2$. What are the eigenvalues of A ?
- (6) A is a 3×3 triangular matrix with diagonal entries 1,2,2. Can you find the eigenvalues? Do you know whether A is diagonalizable?
- (7) A is a 2×2 matrix with eigenvalues 0,1 and corresponding eigenvectors $\langle 1, 3 \rangle$ and $\langle 3, -1 \rangle$. Is A symmetric? What are $\text{Tr}(A)$ and $\det(A)$? How many such A exist? What's $A^{1000} - A$?
- (8) If A is an $m \times n$ matrix with orthonormal columns, prove that the orthogonal projection onto $CS(A)$ is AA^T . Why is it “easy” to find the least-squares solution to $Ax = b$?