MATH 54 S215/S205: REVIEW FOR HILL §1.1–3.6

(1) Let A' be an $n \times (n+1)$ augmented matrix representing a linear system. Show that the n-vector x is a solution to the system if and only if $A' \begin{bmatrix} x \\ -1 \end{bmatrix} = 0$.‘

(2) If A is an $n \times n$ matrix in row echelon form, is $A + I$ in row echelon form? In this case, where are its pivots, and is $A + I$ invertible?

(3) A spreadsheet contains two matrices, A and B, with row and column headings.

A holds prices per kg.: its rows are labeled “Walmart” and “Petco”, and its columns are labeled “cat food” and “duck chow”.

B holds kg. per month quantities: its rows are labeled “cat food” and “duck chow”, and its columns are labeled Jan.–Mar.

Interpret the matrix product $AB \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

(4) Are elementary matrices always square? Always invertible? If E is an elementary matrix and A is a matrix of the same shape, which of A, AE, EA are row-equivalent?

(5) If A and B are invertible matrices, which of the following are invertible?

$$A^2, ABA, A + B, A + I, \begin{bmatrix} 0 & A \\ B & 0 \end{bmatrix}$$

What are $NS(A)$, $RS(A)$, and $CS(A)$?

Show that B is invertible if and only if B^2 is invertible.

(6) What is the inverse of $ABC^{-}1$, and when is it defined?

(7) Suppose $A = [x|y|z]$ is a 3×3 matrix built from column vectors.

Explain why $A \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ is a linear combination of $\{x, y, z\}$ but $[a, b, c]A$ may not be.

If $\begin{bmatrix} a \\ b \\ c \end{bmatrix} \in NS(A)$, find a linear dependence relation for $\{x, y, z\}$.

If $\left\{ \begin{bmatrix} a_i \\ b_i \\ c_i \end{bmatrix} \right\}$ is a basis for $NS(A)$, find a system of equations describing the span of $\{x, y, z\}$.

(8) If $\{Ax, Ay, Az\}$ is a linearly independent set, show $\{x, y, z\}$ is. Show that the rank of A is at least 3.

(9) State the rank-nullity theorem.

(10) Let X be a 2-dimensional vector space, and let B be a basis for X. Is B a vector space? How large, as a set, is B? If $x, y \in B$, and $ax + by = 0$, what can you say about a and b?

(11) When is the solution set for $Ax = b$ a vector space?

Do the 9×9 matrices A satisfying $A^2 = A$ form a vector space?

What about the invertible 6×6 matrices?