## MATH 54 S215/S205: REVIEW FOR BOYCE&DIPRIMA

Note: It may also be a good idea to consult the handout on sections 3.1–7.1.

(1) Solve: $y'' - 4y = 0$.
   Solve: $y'' - 4y = 4e^{2x}$, given $y = xe^{2x}$ is a solution.
   Solve the above equations subject to the initial conditions $y(0) = 0$, $y'(0) = 4$.

(2) When do the solutions of $y'' - 2y' = f(t)$ form a vector space? When they do, what is the general solution? What is a fundamental set?

(3) Show that $W(af_1 + cf_2, bf_1 + df_2) = W(f_1, f_2) \cdot \det \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.
   Assume that $\{f_1, f_2\}$ is a linearly independent set. Why does this equation not prove that $\{af_1 + cf_2, bf_1 + df_2\}$ is linearly independent if and only if $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$?
   Explain why this statement is true anyway.

(4) Consider the ODE $y'' + p(t)y' + q(t)y = 0$, where $p(t)$ and $q(t)$ are continuous functions.
   If $y$ is a solution, $y(t_0) = 0$, $y'(t_0) = 0$; show $y(t) = 0$.
   Explain why $y(t) = \cos^2(t)$ cannot be a solution.
   If $y_1, y_2$ are solutions, $W(y_1, y_2)(t_0) = 0$; show $\{y_1, y_2\}$ is a linearly dependent set.
   Explain why $y_1(t) = t^2 + t + 4$, $y_2(t) = t$ cannot both be solutions.

(5) The system $x' = Ax$ has a spiral-shaped phase portrait. Explain why $A$ cannot be a symmetric matrix.
   The system $x' = Bx$ has a solution of the form $x(t) = \eta e^{\lambda t} + \xi t e^{\lambda t}$. Find $(B - \lambda I)\eta$, $(\lambda I - B)\eta$, and (most importantly) a second, linearly independent solution.

(6) If $\Phi(t)$ is a fundamental matrix for an $n \times n$ system and $t_0$ is some value of $t$, what are $\mathrm{rk}(\Phi(t_0))$ and $\dim NS(\Phi(t_0))$?

(7) Fill in the blank: if $y'' = \lambda y$, $y'' - 9y =$ ______$y$.
   Solve: $y'' - 4y = \cosh(x)$, $y'(0) = 0$, $y'(1) = m$. Does the value of $m$ affect the number of solutions?

(8) $f(x) = x^2$, $0 \leq x < \pi$. Set up one integral each for the coefficients of the sine series and the cosine series (each of period $2\pi$).

(9) If $a_n = \frac{1-\cos(n\pi)}{n}$ and $b_n = \sin(n\pi/2)$, plug in $n = 2m - 1$ and simplify.

(10) Separate: $u_{xx} + u_{yy} + xu = 0$.