Math 128A (Blanchard) Su09: **Quiz 4**, Aug 11, 2009 Solutions

## PROBLEM ONE [4 PTS.]

Use elimination with partial pivoting to turn $\left[\begin{array}{ccc|c} 0 & 2 & -1 & 0 \\ 1 & 1 & -1 & 1 \\ 0 & 4 & 1 & 6 \end{array}\right]$ into an equivalent triangular system. (Using another method is worth partial credit.)

**Solution:** $$\underbrace{\left[\begin{array}{ccc|c} 0 & 2 & -1 & 0 \\ 1 & 1 & -1 & 1 \\ 0 & 4 & 1 & 6 \end{array}\right]}_{|1|>|0|,|0|} \longrightarrow \underbrace{\left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 2 & -1 & 0 \\ 0 & 4 & 1 & 6 \end{array}\right]}_{|4|>|1|,|2|} \longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 4 & 1 & 6 \\ 0 & 2 & -1 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 4 & 1 & 6 \\ 0 & 0 & -\frac{3}{2} & -3 \end{array}\right]$$

## PROBLEM TWO [4 PTS.]

Solve the system from problem 1.

**Solution:** By back-substitution, $x_3 = 2 \implies x_2 = 1 \implies x_1 = 2 \ldots$ so $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$.

## PROBLEM THREE [4 PTS.]

Find an $LU$ factorization for the following matrix. You may use any method. $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

**Solution 1:** If you run the Doolittle or Crout algorithm you can just write down the factors.

**Solution 2:** Assume a Doolittle-style factorization: $A = \begin{bmatrix} 1 & 0 \\ x & 1 \end{bmatrix} \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} = \begin{bmatrix} a & b \\ x & xb + c \end{bmatrix}$.

Then $a = 1$, $b = -1$, $x = 1$, and so $c = 2$.

Or assume a Crout-style factorization: $A = \begin{bmatrix} a & 0 \\ b & c \end{bmatrix} \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & ax \\ bx & bx + c \end{bmatrix}$.

Then $a = 1$, $b = 1$, and so $x = -1$, and so $c = 2$.

**Solution 3:** Elimination transforms $A$ into an upper triangular matrix with one operation: adding $-1$ times row 1 to row 2. So $A$ is the inverse of this operation times its result, $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$.

All three solutions produce the factorizations $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$ or $A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$.

Other methods and answers are possible.