

Three problems, 20 minutes, pencil-and-paper only. Your work is more important than your answer. Good luck!

PROBLEM ONE [4 PTS.]

Find the composite trapezoidal estimates $R_{1,1}$ and $R_{2,1}$ to $\int_0^2 x^4 dx$. Compute the Romberg estimate $R_{1,2} = \frac{4R_{2,1} - R_{1,1}}{3}$. Which estimate came closest to the exact value, 6.4?

PROBLEM TWO [4 PTS.]

Find a C^2 function $r(x)$ and a number s , such that $\int_0^1 \frac{\cos(x)}{x^{1/2}} dx = \int_0^1 r(x) dx + s$. (Pretend that you want to use a composite trapezoidal estimate.)

PROBLEM THREE [4 PTS.]

Find a quadratic polynomial q satisfying the following orthogonality relations on $[0, 1]$ (not $[-1, 1]$):
 $\int_0^1 x^0 q(x) dx = 0, \quad \int_0^1 x^1 q(x) dx = 0.$