

You have 20 minutes to complete this quiz. It's double-sided. You may use your brain and pen/pencil. No other resources are allowed. You will get credit for both answers and work.

PROBLEM ONE [4 PTS.]

The function $f(x) = x^3 - 7$ has one real root. Find an interval which brackets it. Starting with this interval, how many iterations of the bisection method are needed to ensure a relative error of less than 0.25 in approximating the solution?

PROBLEM TWO [4 PTS.]

Find the rate of convergence of $f(h) = 1 - \cos(h)$ as $h \rightarrow 0$. Express your answer in the form $f(h) = O(h^r)$, where r is a number.

PROBLEM THREE [4 PTS.]

Let $g(x) = \frac{2}{3}(x + x^{-2})$. Show that fixed-point iteration, $p_{n+1} = g(p_n)$, converges for any $p_0 \in [1, 2]$.