MATH 128A, SUMMER 2009: PROGRAMMING ASSIGNMENT 3

In this assignment, you will solve a stiff ODE using Euler's method (a general-purpose method) and the implicit trapezoidal rule (an A-stable method). The equation in question models logistic population growth:

$$\begin{cases} y' = y(10 - y) \\ y(0) = 2 \end{cases}$$

The exact solution to this problem is $y(t) = 10/(1 + 4e^{-10t})$.

(1) To run a step of the implicit trapezoidal method, one must solve the following equation for w_{i+1}: $w_{i+1} = w_i + \frac{h}{2}[f(t_{i+1}, w_{i+1}) + f(t_i, w_i)]$.[1]

Write out Newton's method for solving this equation, with the initial guess $w_{i+1}^{(0)} = w_i$. Your formula should depend on $\frac{\partial f}{\partial y}$.

(2) Write `function [ti,wi]=implicit_trap(f,dfdy,a,b,y0,N,tol,max_iterations)` implementing the implicit trapezoidal method. Use Newton's method as above. Here `dfdy` is $\frac{\partial f}{\partial y}$, `N` is the number of time steps, and `tol` and `max_iterations` are constraints for Newton's method.

(You may use the function `newton` which is available in lab or on Justin's web site. If you get it from the latter source, you must also download `fixedpoint.m`.)

(3) Plot the exact solution ($0 \le t \le 4$), the Euler's method estimate with $h = 0.25$, and the implicit trapezoidal estimate with $h = 0.25$, on a common set of axes. For the last of these, set `tol` $= 10^{-4}$ and `max_iterations` $= 20$. (Clearly indicate which curve is which.)

(4) Find the absolute error in estimating $y(1)$ with the implicit trapezoidal rule with $h = 1/8$, $h = 1/16$, $h = 1/32$, and $h = 1/64$. Use the `loglog` function to create a log-log plot of the absolute error versus the step size. Use this to estimate the method's rate of convergence.

Your report should contain all code needed to reproduce your results, your graphs from #3 and #4, and the absolute error values and estimated rate of convergence in #4. Numerical output should be accurate to six significant digits.

Date: Due Wednesday 8/12.

[1] It is possible to use the rule in a predictor-corrector method, but the resulting algorithm will not be A-stable.