UCB Math 128A-2, Summer 2009: Midterm 2
Tuesday 7/28

Name: _____

- No books, notes, or calculators
- Justify all answers
- Time limit is 60 minutes
- **Do not open until instructed to do so.**

Grading

1.	/	20
2.	/	20
3.	/	20
4.	/	20
5.	/	20
	/	100

Scratch/extra work

1. Fill in the blanks below as accurately as possible.

x	0	0.5	+1
$f(x)$	1		3
$f'(x)$			

$\int_0^1 f(x)\,dx \approx$ $\square$

2. Let $T(h)$ denote the composite trapezoidal estimate of $\int_0^1 f(x)\,dx$, with step size h.

   You have measured $T(0.2) = 3.0$ and $T(0.1) = 3.3$. Assume that the composite trapezoidal rule makes error $\int_0^1 f(x)\,dx - T(h) \approx Kh^2$ (for a non-zero constant K).

   (a) Estimate the exact value of $\int_0^1 f(x)\,dx$ as accurately as possible.

   (b) Estimate the absolute error $\left|\int_0^1 f(x)\,dx - T(0.1)\right|$ as accurately as possible.

   (c) Estimate how small h must be for $T(h)$ to approximate $\int_0^1 f(x)dx$ to within 10^{-3}

3. Find the largest possible degree of precision for the following quadrature rule, and the corresponding values of a and b:

$$\int_{-1}^{1} f(x)\,dx \approx af\left(-\frac{5}{6}\right) + bf(0) + af\left(+\frac{5}{6}\right).$$

4. Consider the initial value problem $y'(t) = t^2 + 2y$ $(t \in [0, 4])$ with $y(0) = 0$.

   (a) Fill in the blanks using Euler's method:

t	0.0	1.0	2.0	3.0	4.0
$y(t)$	0.0				

   (b) Find a Lipschitz constant L for the problem.

   (c) Find a formula for y'' in terms of t and y.

   (d) Redo (a) with a Taylor method, order 2:

t	0.0	1.0	2.0	3.0	4.0
$y(t)$	0.0				

5. Find the order of the local truncation error τ_{i+1} in this rule for solving $y' = f(t, y)$:

$$w_{i+1} = w_i + hf(t_i + h, w_i).$$