UCB Math 128A-2, Summer 2009: Midterm 1
Tuesday 7/7

Name: _____

- No books, notes, or calculators
- Justify all answers
- Time limit is 60 minutes
- **Do not open until instructed to do so.**

Grading

1.	/	20
2.	/	20
3.	/	20
4.	/	20
5.	/	20
	/	100

Scratch/extra work

1. (a) Use a third-order Taylor polynomial to estimate $\cos(0.1)$.
   Express the result as a decimal.
   (b) Find a bound for the error in this approximation. Express the result as a fraction.

2. Show that the interval $[0, 1]$ contains a root of $f(x) = 2x^3 - 4x + 1$.

3. Find a quadratic polynomial $P(x)$ such that $P(0) = 0$, $P(1) = 3$, and $P(2) = 4$.

4. Let $f(x) = 2^x$. Compute the following divided differences:

(a) $f[0, 1]$ (b) $f[1, 2]$ (c) $f[0, 1, 2]$

5. (a) Show that the interval $[0, 1]$ contains a fixed point of $g(x) = (1/2)^x$.
   (b) Show that fixed-point iteration $p_{n+1} = (1/2)^{p_n}$ converges for any $p_0 \in [0, 1]$.
   Hint: $\ln(1/2) \approx -0.69$.
   (c) [Bonus] Is this convergence linear?