MATH 128A, SUMMER 2009: HOMEWORK 7

(1) Two multistep methods with step size $h = 0.1$ were used to estimate y_{i+1}:
An Adams-Bashforth 3-step explicit method gave $w_{i+1} = .33$.
An Adams-Moulton 2-step implicit method, using this estimate as a predictor, improved it to .23.
Using ideas from section 5.7, estimate the latter method's local truncation error.

(2) Express this as a system of 1st-order IVPs: $y'' + 2y' + 2y = 0$, $0 \le t \le 9.6$; $y(0) = 1$, $y'(0) = -1$.

(3) Find the absolute error from estimating $y(9.6)$ using `euler`. Use step sizes 0.6, 0.8, and 1.2. The exact solution is $y(t) = e^{-t}\cos(t)$.

(4) The purpose of this problem is to look at a weak stability in a real-life algorithm, and also to motivate one of the steps in the extrapolation algorithm in section 5.8. Consider the following multistep method, which is based on the midpoint method and is this algorithm's foundation:

$$w_{i+1} = w_{i-1} + 2hf(t_i, w_i).$$

(a) Show that this multistep method is weakly stable.
(b) Find all solutions to the homogeneous recurrence $w_{i+1} = w_{i-1}$ (the limit of this rule as $h \to 0$). (Look for solutions of the form $w_i = \lambda^i$. Then take linear combinations.)
(c) In light of (4b), why does figure 1 looks like a child tried to draw teeth on a logarithm curve?
(d) In light of (4b), why can smoothing (estimating y_i with $\frac{1}{4}(w_{i-1} + 2w_i + w_{i+1})$, which is a weighted average of w_i and its immediate neighbors) improve this method's accuracy?

(5) (a) Show that the region of absolute stability for Euler's method is $\{|h\lambda + 1| < 1\}$.
(b) Show that the region of absolute stability for the backward Euler method ($w_{i+1} = w_i + hf(t_{i+1}, w_{i+1})$) is $\{|h\lambda - 1| > 1\}$.
(c) The first-order system from problem 2 is stiff. Compute this (linear) system's eigenvalues. Use this and (5a) to estimate how small h must be for Euler's method to solve this problem stably. Does the backward Euler's method have any such restriction?

Date: Due Thursday 8/06.

FIGURE 1. Figure for problem (4c)