MATH 128A, SUMMER 2009: HOMEWORK 6

Problems 5.3: 1(c), 5.4: 1(c) and 13(c), 5.5: 1(c), modified versions of 5.6: 1(c) and 4:

Use the following methods to solve the problem of 5.2: 1(c) from last week ($y' = 1 + y/t$, $t \in [1, 2]$, $y(1) = 2$,). Put your answers (for each value of t) in a table, rounded to four significant digits (`x.xxx`).

(1) Actual solution: $y = t(\ln t + 2)$;
(2) Taylor method of order 2, $h = 0.25$;
(3) Modified Euler method, $h = 0.25$;
(4) Adams-Bashforth two-step explicit method, $h = 0.25$, with exact starting values;
(5) Adams-Moulton one-step implicit method ($w_{i+1} = w_i + \frac{h}{2}(f(t_{i+1}, w_{i+1}) + f(t_i, w_i))$), $h = 0.25$, with exact starting values;
(6) Adams second-order predictor-corrector method (one-step Adams-Bashforth, i.e. Euler, feeding the above Adams-Moulton formula), $h = 0.25$, with exact starting values;
(7) Runge-Kutta method of order 4 (see `rk4.m`), $h = 0.25$;
(8) Runge-Kutta-Fehlberg method (see `rk4.m`), tol $= 10^{-4}$, $h_{\min} = 0.05$, $h_{\max} = 0.25$;

RK problem: Assume that the differential equation $y' = f(t, y)$ takes the form $y' = g(y)$. (Equations of this form are called *autonomous*.) Find the best choice of coefficients a, b_1, b_2, and c in the following Runge-Kutta method:

$$k_1 = hg(w_i)$$
$$k_2 = hg(w_i + ak_1)$$
$$w_{i+1} = w_i + b_1 k_1 + b_2 k_2$$

What is the order of the resulting method's truncation error?

Multistep Method Problem: Find a_0, a_1, b_0, and b_1 so that the following explicit 2-step method has the highest-order truncation error possible:

$$w_{i+1} = a_1 w_i + a_0 w_{i-1} + h[b_1 f(t_i, w_i) + b_0 f(t_{i-1}, w_{i-1})]$$

Hint: Let $y(t) = c_0 + c_1(t - t_i + c_2(t - t_i)^2 + c_3(t - t_i)^3 + \ldots$. Match as many terms as possible in the power series $y(t_i + h)$ and $a_1 y(t_i) + a_0 y(t_i - h) + h[b_1 y'(t_i) + b_0 y'(t_i - h)]$.

Note: In spite of the good truncation error, the resulting method is unsuitable for all problems.

Date: Due Thursday 7/30.