MATH 128A, SUMMER 2009: HOMEWORK 3 SOLUTIONS

3.1.5a. To minimize the error bound, we should take as our x_i the $(n+1)$ points nearest $x = 8.4$. When $n = 2$ there's a tie; both reasonable answers are shown below.

```
octave:1> format long
octave:2> neville(8.4,[8.3,8.6], [17.56492, 18.50515])                       %n=1
ans =  17.8783300000000
octave:3> neville(8.4,[8.3,8.6,8.1], [17.56492, 18.50515, 16.94410])         %n=2
ans =  17.8771300000000
octave:4> neville(8.4,[8.3,8.6,8.7], [17.56492, 18.50515, 18.82091])         %n=2
ans =  17.8771550000000
octave:5> neville(8.4,[8.3,8.6,8.1,8.7], [17.56492, 18.50515, 16.94410, 18.82091]) %n=3
ans =  17.8771425000000
```

3.1.19c. There are many ways to find the polynomial. A correct answer, when expanded with a calculator, should come out to $P(x) = 0.1970056667x^3 - 1.06259055x^2 + 2.532453189x - 1.666868305$. The error, for $x \in [1, 1.4]$, is of the form $\frac{\ln^{(4)}(\xi(x))}{4!}(x-1)(x-1.1)(x-1.3)(x-1.4)$. Since $\ln^{(4)}(\xi) = (-3)(-2)(-1)x^{-4} = -6\xi^{-4}$, it's at worst $-6(1)^{-4} = -6$. $|(x-1)(x-1.1)(x-1.3)(x-1.4)|$ is at most $4 \cdot 10^{-4}$, attained when $x = 1.2$. So $P(x)$ approximates $\ln(x)$ to within 10^{-4} on $[1, 1.4]$.

Extra. $f[0,1,2] = \frac{f[1,2]-f[0,1]}{2-0} = \frac{\frac{f[2]-f[1]}{2-1} - \frac{f[1]-f[0]}{1-0}}{2} = \frac{(8-1)-(1-0)}{2} = 3.$

3.2.11. (a) Calculation shows $P(x) = f(x)$ and $Q(x) = f(x)$ for $x = -2, -1, 0, 1, 2$.
(b) Both polynomials are equal (to $x^3 - 3x + 1$), just written in a different form.

3.3.1a. Construct a divided-difference table as follows.

$$
\begin{array}{r|lccc}
x_0 = 8.3 & f[x_0] = f(8.3) = 17.56492 & & & \\
 & & f[x_0, z_0] = f'(8.3) = 3.116256 & & \\
z_0 = 8.3 & f[z_0] = f(8.3) = 17.56492 & & 0.0594800 & \\
 & & f[z_0, x_1] = \cdots\cdots = 3.134100 & & -0.002022222 \\
x_1 = 8.6 & f[x_1] = f(8.6) = 18.50515 & & 0.0588733 & \\
 & & f[x_1, z_1] = f'(8.3) = 3.151762 & & \\
z_1 = 8.6 & f[z_1] = f(8.6) = 18.50515 & & &
\end{array}
$$

(See, e.g., Table 3.13 in the text.) The coefficients for the Newton form of the approximating polynomial are on the top edge of this table: $H(x) = 17.56492 + 3.116256(x-8.3) + 0.0594800(x-8.3)^2 - 0.002022222(x-8.3)^2(x-8.6)$.

Date: Due Thursday 7/09.