

Section 101 of Math 54, Summer 2007

Solutions of Sample Quiz 3

1. (5 points) Consider the matrix

$$A = \begin{bmatrix} 0 & 7 & 0 & 5 \\ 3 & 1 & 2 & 0 \\ -3 & 2 & 0 & -1 \\ 0 & 4 & 0 & 3 \end{bmatrix}.$$

Evaluate the determinant of A by cofactor expansion and/or elementary row operations.

Solution. We apply the cofactor expansions on the third column and the first column of the new matrix, that is

$$\det A = 2 \times (-1)^{2+3} \begin{vmatrix} 0 & 7 & 5 \\ -3 & 2 & -1 \\ 0 & 4 & 3 \end{vmatrix} = (-2) \times (-3) \times (-1)^{1+2} \begin{vmatrix} 7 & 5 \\ 4 & 3 \end{vmatrix}.$$

Hence

$$\det A = (-6) \times (7 \times 3 - 4 \times 5) = -6.$$

2. (5 points) Suppose A , B and C are $n \times n$ invertible matrices and

$$\det(A^{-1}) = 2, \det B = 3, \det(ACB) = 6.$$

Evaluate the determinant of C .

Solution. It follows from

$$\det A \det(A^{-1}) = \det(AA^{-1}) = \det I = 1$$

that

$$\det A = \frac{1}{\det(A^{-1})} = \frac{1}{2}.$$

Since $6 = \det(ACB) = \det A \det C \det B$, we get

$$\det C = \frac{6}{\det A \det B} = \frac{6}{3/2} = 4.$$