MATH 53, Fall 2006
Worksheet, November 20, 2006

GREEN'S THEOREM

Problem 1. Compute

$$\oint_C xy^2 dx + x^3 dy,$$

where C is the rectangle with vertices $(0,0)$, $(2,0)$, $(2,3)$ and $(0,3)$.

Problem 2. Use Green's theorem to prove that if $\mathbf{F} = P\mathbf{i} + Q\mathbf{j}$ is a vector field on an open simply-connected region, such that P and Q have continuous first-order derivatives and

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

then $\mathbf{F}$ is conservative.

Problem 3. Use Green's theorem to evaluate the integral

$$\int_C e^y dx + 2xe^y dy,$$

where C is the square with sides $x = 0$, $x = 1$, $y = 0$ and $y = 1$.

Problem 4. Let $\mathbf{F}(x, y) = (-y\mathbf{i} + x\mathbf{j})/(x^2 + y^2)$. Show that $\int_C \mathbf{F} \cdot d\mathbf{r} = 2\pi$ for every positively oriented simple closed path that encloses the origin.

Problem 5. Find the work done by a force $\mathbf{F}(x, y) = x(x + y)\mathbf{i} + xy^2\mathbf{j}$ in moving a particle from the origin along the x-axis to $(1, 0)$, then along the line segment to $(0, 1)$, and then back to the origin along the y-axis. (Hint: Use Green's theorem.)