MATH 53, Fall 2006
Worksheet, November 16, 2006

Problem 1. Prove that the vector field

$$\mathbf{F}(x, y) = (x + y)\mathbf{i} + (x - 5)\mathbf{j}$$

is conservative. Is the field $\mathbf{F}(x, y) = (x - y)\mathbf{i} + (x - 5)\mathbf{j}$ conservative as well?

Problem 2. If $\mathbf{F}$ is the first field from Problem 1, find a function $f(x, y)$, such that $\bigtriangledown f = \mathbf{F}$. Compute

$$\int_C \mathbf{F} \cdot d\mathbf{r},$$

where $\mathbf{r}(t) = e^t \sin t\mathbf{i} + e^t \cos t\mathbf{j}$ for $0 \leq t \leq \pi$.

Problem 3. Show that the force

$$\mathbf{F}(x, y) = \frac{y^2}{x^2}\mathbf{i} - \frac{2y}{x}\mathbf{j}$$

is conservative and find the work done by that force in moving a particle from $P(1, 1)$ to $Q(4, -2)$.

Problem 4. Show that if the vector field $\mathbf{F} = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$ is conservative and P, Q, R have continuous first-order partial derivatives then

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}, \frac{\partial P}{\partial z} = \frac{\partial R}{\partial x}, \frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y}.$$

Problem 5. Consider the regions (a) $\{(x, y) : x, y > 0\}$, (b) $\{(x, y) : 1 \leq x^2+y^2 \leq 5\}$, (c) $\{(x, y) : x \neq 0\}$ and (d) $\{(x, y) : (x, y) \neq (0, 0)\}$. Which of the regions (a), (b), (c) and (d) are (i) open, (ii) connected, (iii) simply connected?

Problem 6. Find the work done by the gravitational field

$$\vec{F}(\vec{x}) = -\frac{mMG}{|\vec{x}|^3}\vec{x}.$$

(Hint: Use the fact that $\vec{F}$ is a conservative force with potential function)