

Volume of solid of revolution:

region bounded by . . .	about x -axis	about y -axis
$y = f(x), y = 0, x = 0, x = A$	$\pi \int_0^A (f(x))^2 dx$	$2\pi \int_0^A x f(x) dx$
$x = g(y), x = 0, y = 0, y = B$	$2\pi \int_0^B x g(y) dy$	$\pi \int_0^B (g(y))^2 dy$

region below $y = x^2$ from $(0, 0)$ to $(A, B) = (A, A^2) = (\sqrt{B}, B)$

about y -axis: $2\pi \int_0^A x(x^2) dx = \frac{2\pi}{4} A^4$

about x -axis: $\pi \int_0^A (x^2)^2 dx = \frac{\pi}{5} A^5;$

region to left of $y = x^2$ from $(0, 0)$ to (A, B)

about y -axis: $\pi \int_0^B (\sqrt{y})^2 dy = \frac{\pi}{2} B^2$

about x -axis: $2\pi \int_0^B y(\sqrt{y}) dy = 2\frac{\pi}{5/2} B^{5/2}$

Problem 26: want region to left of $y = x^2$ (bounded by $x = 0$ and $y = 4$), rotated about y -axis:

Using the 1st of the above formulas (with $A = 2, B = 4$), we get Volume = $\frac{2\pi}{4} A^4 \stackrel{A=2}{=} 8\pi$.

Alternatively, can use 3rd formula (which gives $\frac{\pi}{2} B^2 \stackrel{B=4}{=} 8\pi$) and subtract this from the volume of the cylinder generated by rotating the rectangle with corner at $(2, 4)$ which is $\pi B A^2 = 16\pi$