Quiz 6 Solutions

1) $y' = \sin(ty) \quad -10 \le t \le 10 \quad y(-10) = 1 \quad (*)$

$f(t,y) = \sin(ty)$

$\left|\frac{\partial f}{\partial y}(t,y)\right| = |t\cos(ty)| = |t|\,|\cos(ty)| \le 10 \cdot 1 = 10$

Hence, by thm 5.3 f is Lipschitz in y
=> by thm 5.4 that there is a unique solution to $(*)$.

2) $\tau_{i+1}(h) = \frac{y(t_{i+1}) - y(t_i)}{h} - f(t_i, y(t_i)) - \frac{h}{2} f'(t_i, y(t_i))$

$y(t_{i+1}) = y(t_i + h) = y(t_i) + h y'(t_i) + \frac{h^2}{2} y''(t_i) + \frac{h^3}{6} y'''(\xi_i)$

where ξ_i is between t_i and $t_i + h$

$\Rightarrow \frac{y(t_{i+1}) - y(t_i)}{h} = y'(t_i) + \frac{h}{2} y''(t_i) + O(h^2)$

$\Rightarrow \frac{y(t_{i+1}) - y(t_i)}{h} - y'(t_i) - \frac{h}{2} y''(t_i) = O(h^2)$

But $y' = f(t,y) \Rightarrow y'' = f'(t,y)$

$\Rightarrow \tau_{i+1}(h) = \frac{y(t_{i+1}) - y(t_i)}{h} - f(t_i, y(t_i)) - \frac{h}{2} f'(t_i, y(t_i))$
$= O(h^2)$

Hence, $\tau_{i+1}(h) = O(h^2)$.