

1. Find the sum of the series $\sum_{n=1}^{\infty} 7 \left(\frac{2}{3}\right)^n$

2. (a) In the expression $\sum_{n=0}^{\infty} 5(x-3)^n$, if you replace x by any real number what you get is an honest geometric series like we have seen. For which values of x does the resulting series converge?
 - (b) Suppose your 7-year-old niece asks you what is meant by the expression

$$\frac{5}{4-x} = 5 + 5(x-3) + 5(x-3)^2 + 5(x-3)^3 + \dots \quad \text{for } |x-3| < 1$$

What will you tell her?

1. Consider the sequence described by $a_n = \left(\frac{1}{n} - \frac{1}{n+1}\right)$, for $n \geq 1$.
 - (a) Write out a few of the partial sums, s_1 , s_2 , s_3 and s_4 for the series $\sum_{n=1}^{\infty} a_n$.
 - (b) Find a simplified expression for s_n .
 - (c) Does the series $\sum_{n=1}^{\infty} a_n$ converge? If not, why not? If it does, what does it converge to?

2. I'm very interested in the series:

$$\sum_{n=1}^{\infty} \left[\left(\frac{1}{7}\right)^n + 15 \left(\frac{1}{n} - \frac{1}{n+1}\right) \right]$$

Unfortunately, it's only the first day of the series unit, and we have very few theorems about series. If wishes were ponies, and you could *invent* any theorems about series you wanted, what would be some reasonable and useful theorems for understanding the expression above. Try to state your invented theorems as carefully as possible, and explain to your group how they would help understand the series above.